



Rewarding Learning

ADVANCED

General Certificate of Education

2016

---

# Mathematics

Assessment Unit C4

*assessing*

Module C4: Core Mathematics 4

**MV18**

[AMC41]

**WEDNESDAY 8 JUNE, MORNING**

---

## **Time**

1 hour 30 minutes, plus your additional time allowance.

## **Instructions to Candidates**

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

## Information for Candidates

The total mark for this paper is 75

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Find

$$\int \frac{x+4}{x(2-x)} dx \quad [7 \text{ marks}]$$

**2** Relative to a fixed origin O, points A and B have coordinates (1, 3, 5) and (−2, 7, 4) respectively. A laser beam is directed from A towards a hole at B.

**(i)** Find the distance AB. [2 marks]

The path of the laser beam can be modelled by the equation of a straight line.

**(ii)** Find the vector equation of this line. [4 marks]

A target is placed at the point with coordinates (16, −17, 9).

**(iii)** Find whether or not the laser will hit the target.  
[2 marks]

**3** Solve the equation

$$\sin 2\theta = \cot \theta$$

where  $-\pi \leq \theta \leq \pi$  [8 marks]

**4** Given the functions

$$f(x) = \frac{5}{x+1} \quad x \neq -1$$

$$g(x) = 2x + 3 \quad x \geq 0$$

$$h(x) = x^2 \quad x \geq 0$$

(i) Find the composite function  $fg(x)$ . [2 marks]

(ii) Find the inverse function  $f^{-1}(x)$  stating its domain.  
[5 marks]

(iii) Sketch the graph of  $y = h(x)$ . [1 mark]

(iv) Express in terms of  $f$ ,  $g$  and/or  $h$ :

(a)  $x \rightarrow (2x + 3)^2$  [1 mark]

(b)  $x \rightarrow \sqrt{\frac{5}{x+1}}$  [1 mark]

(c)  $x \rightarrow 4x + 9$  [1 mark]

5 (i) For the equation

$$x^3 - 3x^2y = 4$$

use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{x - 2y}{x} \quad [5 \text{ marks}]$$

(ii) Hence find the stationary point on the curve

$$x^3 - 3x^2y = 4$$

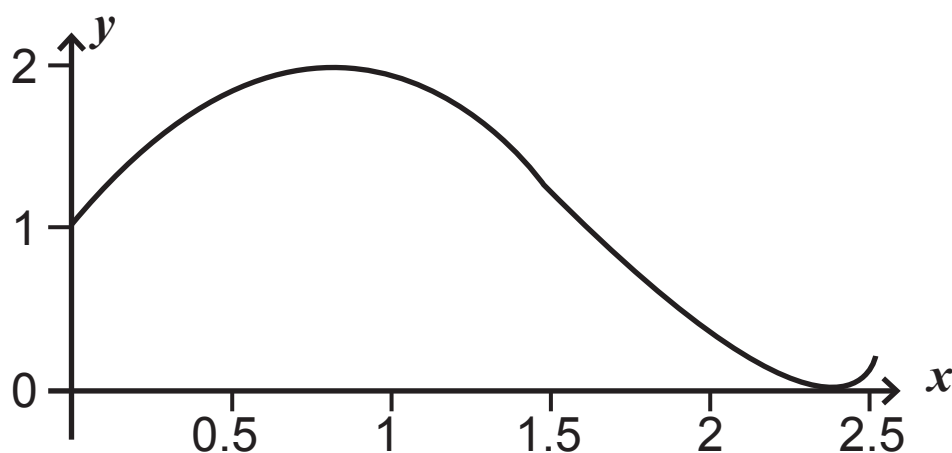
and determine its nature. [9 marks]

6 Given that  $\sin(x - \theta) = 3 \cos(x + \theta)$

prove that  $\tan x \equiv \frac{3 + \tan \theta}{1 + 3 \tan \theta}$  [7 marks]

7 Fig. 1 below shows a sketch of the curve

$$y = 1 + \sin 2x$$



**Fig. 1**

The curved surface area of a bottle stopper can be modelled by rotating the curve

$$y = 1 + \sin 2x$$

between  $x = 0$  and  $x = \frac{3\pi}{4}$  through  $2\pi$  radians about the  $x$ -axis.

Find the exact volume of the bottle stopper. [10 marks]

- 8 A curve passes through the point (1, 2) and its gradient function is given by

$$\frac{x \ln x}{e^{2y}}$$

Find the equation of the curve. [10 marks]

---

**THIS IS THE END OF THE QUESTION PAPER**

---

