Rewarding Learning

ADVANCED
General Certificate of Education
2019

Centre Number

Candidate Number

Mathematics

Assessment Unit C4
assessing
Module C4:
Core Mathematics 4

MV18

[AMC41]
WEDNESDAY 5 JUNE, MORNING

Time

1 hour 30 minutes, plus your additional time allowance.

Instructions to Candidates

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer **all eight** questions in the spaces provided.
Do not write on blank pages.
Complete in black ink only.
Questions which require drawing or sketching should be completed using an H.B. pencil.
All working should be clearly shown in the spaces provided.
Marks may be awarded for partially correct solutions.
Answers without working may not gain full credit.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

Information for Candidates

The total mark for this paper is 75

Figures in brackets printed at the end of each question indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

1 The functions f and g are defined by:

$f(x) = x^2$
$g(x) = x + 2$

(i) Find the composite function $fg(x)$. [2 marks]

(ii) Find the composite function gf(x). [1 mark]

(iii) Hence find the value of x for which $\mathrm{fg}(x) = \mathrm{gf}(x)$.
[2 marks]

2 **Fig. 1** below shows part of the curve

$y = 6 - 2\sqrt{x}$

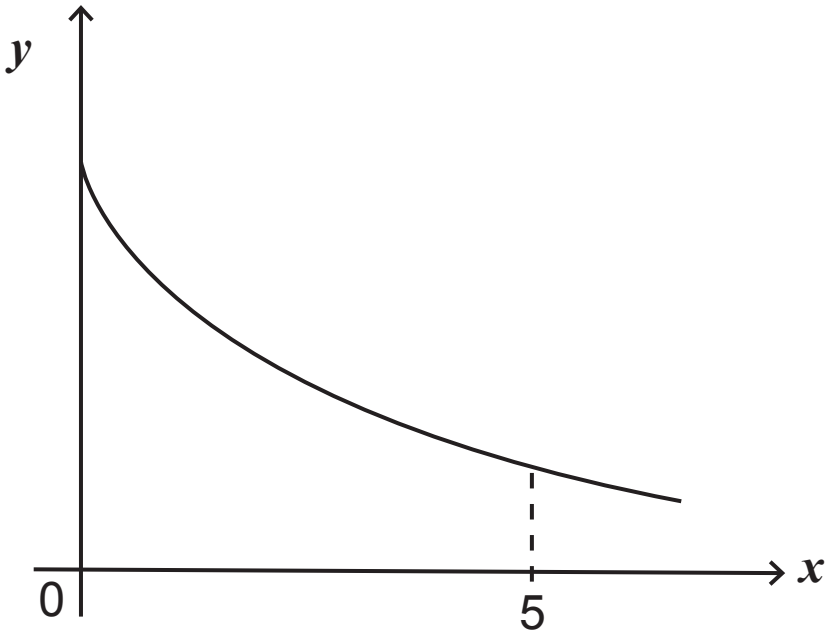

Fig. 1

A plastic funnel can be modelled by rotating this curve between $x = 0$ and $x = 5$ through 360° about the x-axis.

Find the volume that the funnel can hold. [6 marks]

3 An aircraft flies in a straight line from A to B. The position vectors of A and B, relative to a fixed origin O, are

$$\begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix} \text{ and } \begin{pmatrix} 5 \\ 6 \\ 5 \end{pmatrix}$$

respectively.

(i) Find the distance travelled between A and B.
[2 marks]

(ii) Find a vector equation of the line AB. [4 marks]

At B the aircraft changes course and flies towards point C whose position vector is $\begin{pmatrix} 3 \\ 9 \\ 4 \end{pmatrix}$

(iii) Find, in degrees, the angle ABC. [5 marks]

4 **(a)** Find [4 marks]

$$\int (3x + 1)e^{2x}\,dx$$

(b) (i) Prove that [6 marks]

$$\frac{1}{\sin 2x} + \cot 2x \equiv \cot x$$

(ii) Hence find [2 marks]

$$\int \frac{1}{\sin 2x} + \cot 2x \, dx$$

5 **(i)** Using partial fractions, show that [4 marks]

$$\frac{1}{(2x+1)(x+1)} = \frac{2}{(2x+1)} - \frac{1}{(x+1)}$$

(ii) When it is recharged from empty, the voltage, V volts, of a rechargeable battery can be modelled by the equation

$$(2t + 1)\frac{dV}{dt} = \frac{k}{t+1}$$

where t is the number of hours for which the battery is charged and k is a positive constant.
Initially $V = 0$
When $t = 2$, $V = 2.36$
The battery company recommends that a battery needs 24 hours to be fully charged.

Find V when the battery is fully charged. [10 marks]

6 A curve has equation

$$x^2 - 4xy + 2y^2 + 18 = 0$$

(i) Show that [5 marks]

$$\frac{dy}{dx} = \frac{x - 2y}{2x - 2y}$$

(ii) Hence find the coordinates of the stationary points of this curve. [5 marks]

7 Solve the equation

$7\sin\theta + 24\cos\theta = 8$ where $0° \leqslant \theta \leqslant 360°$ [7 marks]

8 The metal blade of a scalpel can be modelled as the area enclosed by the curve

$y = (\sin x - 2\cos x)^2$

the x and y-axes and the line $x = \frac{\pi}{4}$, as shown in **Fig. 2** below.

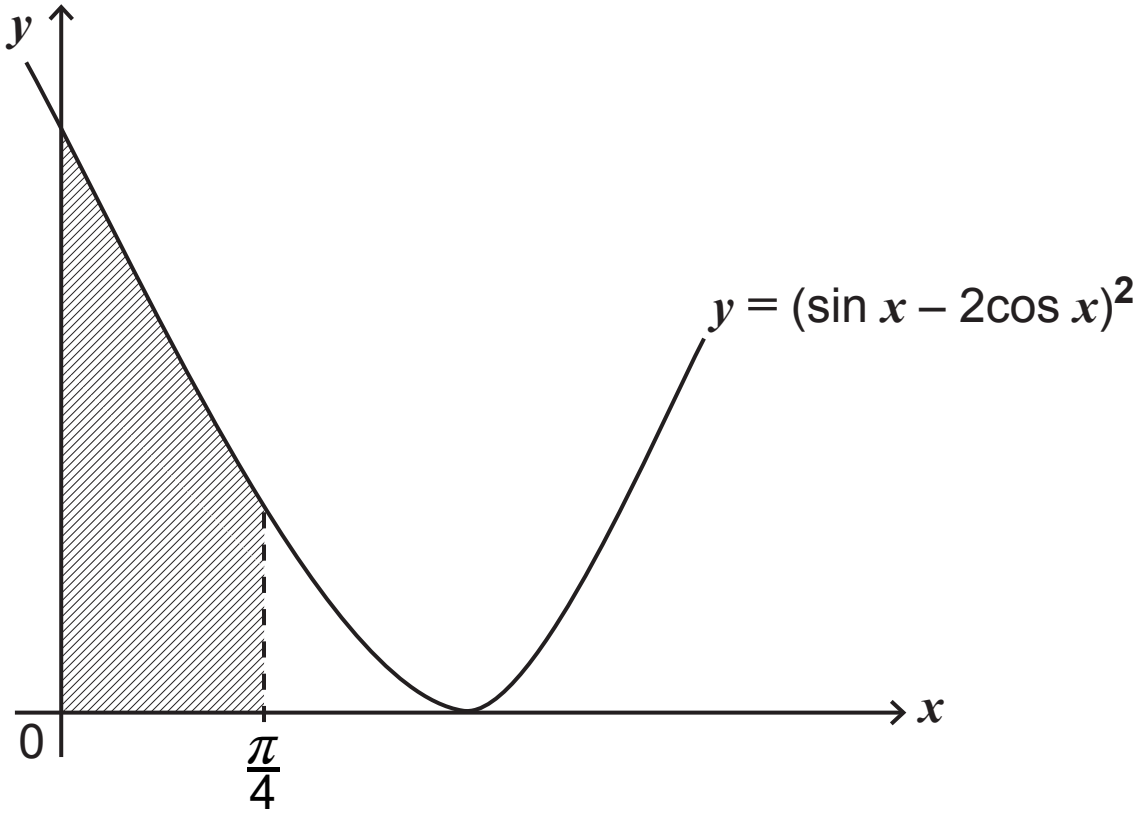

Fig. 2

Find the **exact** area of one face of the blade. [10 marks]

This is the end of the question paper

For Examiner's use only	
Question Number	Marks
1	
2	
3	
4	
5	
6	
7	
8	
Total Marks	

Examiner Number