



Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2011

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]



WEDNESDAY 19 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$



Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 The matrix \mathbf{A} is given by

$$\mathbf{A} = \begin{pmatrix} 7 & -4 \\ -2 & 5 \end{pmatrix}$$

(i) Show that the eigenvalues of \mathbf{A} are 3 and 9 [5]

(ii) Find a **unit** eigenvector corresponding to the eigenvalue 9 [4]

2 Two circles have equations

$$x^2 + y^2 + 2x - 6y + 8 = 0$$

$$x^2 + y^2 - 4x - 28 = 0$$

(i) Find the point where these circles meet. [8]

(ii) Determine whether the circles touch internally or externally. [4]

- 3 (a) Explain why the set $\{1, 2, 3, 4, 5, 6, 7\}$ cannot form a group under **multiplication** modulo 8 [3]

- (b) (i) Copy and complete the group table for **addition** modulo 8

	0	1	2	3	4	5	6	7
0	0	1	2	3	4	5	6	7
1	1	2	3	4	5	6	7	0
2	2	3	4	5	6	7	0	1
3	3	4	5					
4	4	5	6					
5	5	6	7					
6	6	7	0					
7	7	0	1					

[5]

- (ii) Using the group table in (i), or otherwise, write down the two values of x which satisfy

$$x^3 = x$$

[2]

- (iii) For this group, write down a subgroup of order 4

[3]

- 4 (a) Describe fully the transformation given by the matrix

$$\mathbf{M} = \begin{pmatrix} -\frac{3}{5} & \frac{4}{5} \\ \frac{4}{5} & \frac{3}{5} \end{pmatrix}$$

[5]

- (b) The set of points which form the circle

$$x^2 + y^2 = 25$$

is mapped under a transformation given by the matrix

$$\mathbf{N} = \begin{pmatrix} 3 & -2 \\ 2 & -1 \end{pmatrix}$$

Show that the equation of the curve formed by the image points is

$$5X^2 + 13Y^2 - 16XY = 25$$

[8]

5 A matrix \mathbf{M} is given by

$$\mathbf{M} = \begin{pmatrix} 3 & 2 & a \\ 1 & -2 & -1 \\ a & 0 & 3 \end{pmatrix}$$

(i) Find, in terms of a , the determinant of \mathbf{M} . [3]

A system of linear equations is given by

$$3x + 2y + az = 7$$

$$x - 2y - z = 1$$

$$ax + 3z = 11$$

(ii) Find the values of a for which the system has a unique solution. [3]

(iii) If $a = 1$, find the inverse of \mathbf{M} . [6]

(iv) Hence, for $a = 1$ find the unique solution of the system of equations. [3]

6 (a) Find the complex roots of the equation

$$2z^2 - 2iz - 5 = 0 \quad [4]$$

(b) (i) Sketch, on an Argand diagram, the locus of those points w which satisfy

$$|w - 3| = 5 \quad [3]$$

(ii) On the same diagram, shade the region which represents the locus of those points w which satisfy

$$|w - 3| \leq 5$$

and

$$\frac{\pi}{6} \leq \arg(w - 3) \leq \frac{\pi}{4} \quad [6]$$

THIS IS THE END OF THE QUESTION PAPER

