Rewarding Learning

**ADVANCED**
**General Certificate of Education**
**January 2011**

# Mathematics

## Assessment Unit C4

*assessing*

## Module C4: Core Mathematics 4

**[AMC41]**

**FRIDAY 28 JANUARY, MORNING**

# MARK SCHEME

| | AVAILABLE MARKS |
|---|---|

**1** **(a)** **(i)** $\dfrac{1}{(x+1)(x-1)} = \dfrac{A}{(x+1)} + \dfrac{B}{(x-1)}$ MW1

$1 = A\ (x-1) + B\,(x+1)$ MW1

Let $x = 1$ $\quad 1 = 2B \Rightarrow B = \dfrac{1}{2}$ MW1

$x = -1$ $\quad 1 = 2A \Rightarrow A = -\dfrac{1}{2}$ W1

**(ii)** $\dfrac{1}{2}\int \dfrac{1}{(x-1)} - \dfrac{1}{(x+1)}\ dx$ MW1

$= \dfrac{1}{2}\ln|x-1| - \dfrac{1}{2}\ln|x+1| + c$ MW3

**(b)** $\int x\cos x\ dx$

$= x\sin x - \int \sin x\ dx$ M1W2

$= x\sin x + \cos x + c$ W2 13

| | | | AVAILABLE MARKS |
|---|---|---|---|
| **2** | Using dot product | | |
| | $\mathbf{p} \bullet \mathbf{q} = \lvert \mathbf{p} \rvert \lvert \mathbf{q} \rvert \cos\theta$ | M1 | |
| | $(\mathbf{i} - \mathbf{j} - 2\mathbf{k}) \bullet (4\mathbf{i} + 5\mathbf{j} + \mathbf{k})$ | M1 | |
| | $= 4 - 5 - 2 = -3$ | W1 | |
| | $\lvert \mathbf{i} - \mathbf{j} - 2\mathbf{k} \rvert = \sqrt{1^2 + 1^2 + 2^2} = \sqrt{6}$ | M1W1 | |
| | $\lvert 4\mathbf{i} + 5\mathbf{j} + \mathbf{k} \rvert = \sqrt{4^2 + 5^2 + 1^2} = \sqrt{42}$ | MW1 | |
| | $-3 = \sqrt{6}\ \sqrt{42}\ \cos\theta$ | | |
| | $\dfrac{-3}{\sqrt{252}} = \cos\theta$ | | |
| | $\theta = 100.89°$ (101°) | W1 | 7 |
| **3** | $\cos x = 2[\sin x \cos 60° + \cos x \sin 60°]$ | M1W1 | |
| | $\cos x = 2\left[\sin x \dfrac{1}{2} + \cos x \sqrt{\dfrac{3}{2}}\right]$ | MW1 | |
| | $\cos x = \sin x + \sqrt{3}\cos x$ | | |
| | $\left(1 - \sqrt{3}\right)\cos x = \sin x$ | | |
| | $1 - \sqrt{3} = \tan x$ | M1W1 | |
| | $x = -36.2°$ or $144°$ | MW2 | 7 |

| | | AVAILABLE MARKS |
|---|---|---|
| **4 (a) (i)** $1 + \cot^2\theta = \operatorname{cosec}^2\theta$ | M1 | |
| $1 + x^2 = y^2$ | W1 | |
| **(ii)** $y^2 = 1 + x^2$ | | |
| $2y\,\frac{dy}{dx} = 2x$ | M2W2 | |
| $\frac{dy}{dx} = \frac{x}{y}$ | | |
| $y^2 = 1 + 1$ | | |
| $= 2$ | | |
| $y = \pm\sqrt{2}$ | MW2 | |
| $\frac{dy}{dx} = \frac{1}{\pm\sqrt{2}}$ | MW1 | 9 |
| **5 (i)** LHS $\tan x \sec^4 x = \tan x \sec^2 x \sec^2 x$ | MW1 | |
| $= \tan x\,(1 + \tan^2 x)\sec^2 x$ | M1 | |
| $= \tan x \sec^2 x + \tan^3 x \sec^2 x$ | W1 | |
| $=$ RHS | | |
| **(ii)** $\int_0^{\frac{\pi}{4}} \tan x \sec^2 x + \tan^3 x \sec^2 x \; dx$ | MW1 | |
| $u = \tan x \quad \frac{du}{dx} = \sec^2 x$ | MW1 | |
| $\int_0^{\frac{\pi}{4}} (\tan x + \tan^3 x)\sec^2 x \; dx$ | | |
| $\int (u + u^3)\sec^2 x \; \frac{du}{\sec^2 x}$ | M1W1 | |
| $= \left[\frac{u^2}{2} + \frac{u^4}{4}\right]$ | MW1 | |
| $= \left[\frac{\tan^2 x}{2} + \frac{\tan^4 x}{4}\right]_0^{\frac{\pi}{4}}$ | M1 | |
| $= \frac{3}{4}$ | MW1 | 10 |

| | | | AVAILABLE MARKS |
|---|---|---|---|
| **6** | $(1+x^2)\dfrac{dy}{dx} = x(1+y)$ | | |
| | $\displaystyle\int \frac{dy}{1+y} = \int \frac{x\,dx}{(1+x^2)}$ | M2W1 | |
| | $\ln \mid 1+y \mid = \frac{1}{2} \ln \mid 1+x^2 \mid + c$ | W2 | |
| | $x = 0$ when $y = 0 \;\therefore\; c = 0$ | M1W1 | |
| | $\ln \mid 1+y \mid = \frac{1}{2} \ln \mid 1+x^2 \mid$ | | |
| | $1+y = \sqrt{1+x^2}$ | M2W1 | |
| | $y = \sqrt{1+x^2} - 1$ | W1 | 11 |
| **7** | Volume $= \displaystyle\int_0^9 \pi y^2 \, dx$ | M2W1 | |
| | $= \pi \displaystyle\int_0^9 e^{\frac{x}{3}} \, dx$ | W1 | |
| | $= \pi \left[ 3e^{\frac{x}{3}} \, dx \right]_0^9$ | MW1 | |
| | $= 3\pi (e^3 - 1)$ | W2 | 7 |

| | AVAILABLE MARKS |
|---|---|

**8** **(i)** $y = \sin x$

$x = \sin^{-1} y$ MW1

$f^{-1} : x \rightarrow \sin^{-1} x$ MW1

domain $-1 \leq x \leq 1$ W1

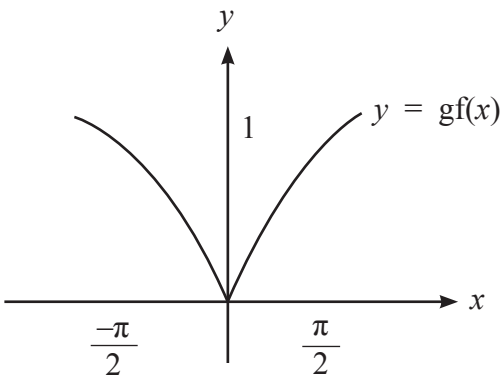
range $-\frac{\pi}{2} \leq f^{-1}(x) \leq \frac{\pi}{2}$ W1

**(ii)** $x \rightarrow \sin x \rightarrow |\sin x|$ M1W1

$gf : x \rightarrow |\sin x|$

range $0 \leq gf(x) \leq 1$ W2

**(iii)**




MW3 11

**Total** **75**