Rewarding Learning

ADVANCED
General Certificate of Education
January 2013

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

[AMC31]

WEDNESDAY 23 JANUARY, MORNING

AMC31

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A radioactive substance decays at a rate which can be modelled by the equation

$$B = 5000e^{-0.007t}$$

where B is the number of particles of the substance remaining at any time t seconds.

Find the value of t when 3000 particles remain. [4]

2 Solve

$$\operatorname{cosec}(\theta + 40°) = 5$$

where $-180° \leqslant \theta \leqslant 180°$ [4]

3 **(i)** Show that the expression

$$\frac{x-1}{3x+5} \div \frac{x^2+3x-4}{9x^2-25}$$

simplifies to

$$\frac{3x-5}{x+4}$$ [4]

(ii) Using the result of **(i)**, find

$$\frac{d}{dx}\left(\frac{x-1}{3x+5} \div \frac{x^2+3x-4}{9x^2-25}\right)$$

giving your answer in its simplest form. [5]

4 **(a)** The reed of a musical instrument can be modelled by the area under the curve

$$y = \frac{1}{x^4 - 1}$$

between the lines $x = 2$ and $x = 4$ and the x-axis, as shown in **Fig. 1** below.

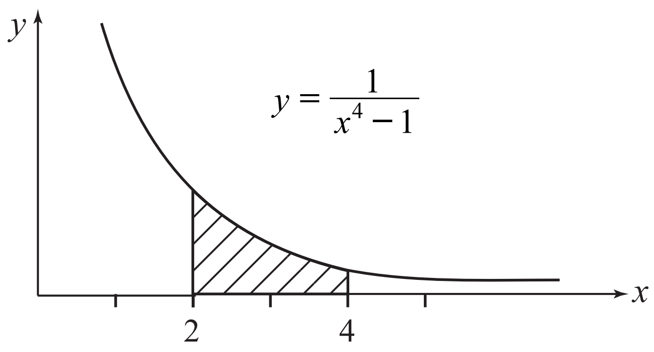

Fig. 1

Use Simpson's rule with 4 strips to find an approximation for the area of the reed,

$$\int_2^4 \frac{1}{x^4 - 1}\,dx$$

[6]

(b) Find

$$\int \left(\frac{7}{x} - \operatorname{cosec}^2 3x - 6x\right) dx$$ [4]

5 **(i)** Solve

$$\ln x - 1 = 0$$ [2]

(ii) Sketch the graph of

$$y = |\ln x - 1|$$ [4]

(iii) Find the **exact** solutions of

$$|\ln x - 1| = 2$$ [5]

6 **(i)** Prove the identity

$$\sin\theta + \cos\theta\cot\theta \equiv \operatorname{cosec}\theta$$ [4]

(ii) Hence, or otherwise, find a Cartesian equation of the parametric curve

$$x = 2 + \cos t \qquad y = \sin t + \cos t\cot t$$ [5]

(iii) Write down the equation of the horizontal axis of symmetry of this curve. [1]

7 Find the equation of the tangent to the curve

$$y = \mathrm{e}^{3x}\cot\left(\frac{1}{2}x + \frac{\pi}{4}\right)$$

at the point where $x = 0$ [8]

8 **(i)** Express

$$\frac{2+7x}{(2-x)(2+3x)}$$

in partial fractions. [5]

(ii) Find the first three terms in the binomial expression of

$$(2+kx)^{-1}$$ [7]

(iii) Hence, using the results from parts **(i)** and **(ii)**, find the first three terms in the binomial expansion of

$$\frac{2+7x}{(2-x)(2+3x)}$$ [7]

THIS IS THE END OF THE QUESTION PAPER