Rewarding Learning

ADVANCED
General Certificate of Education
January 2013

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

AMC41

FRIDAY 25 JANUARY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the coordinates of the point on the curve given parametrically by

$$x = t^2 + 1 \qquad\qquad y = 2t^3$$

at which the gradient is 6 [6]

2 The vertices of a triangle ABC are: A (2, 4, 5)
B (3, 3, –1)
C (0, 1, 4)

Find:

(i) $\overrightarrow{BA}$ and $\overrightarrow{BC}$ [3]

(ii) $|\overrightarrow{BA}|$ and $|\overrightarrow{BC}|$ [3]

(iii) the angle ABC. [3]

3 **(i)** Rewrite

$$5 \sin x - 12 \cos x$$

in the form $R \sin (x - \alpha)$ where R is an integer and $0° \leqslant \alpha \leqslant 90°$ [3]

(ii) Hence solve the equation

$$5 \sin x - 12 \cos x = 7 \qquad\qquad 0° \leqslant x \leqslant 360°$$ [4]

4 **(i)** Given that

$$(x + y)^3 = 2x$$

use implicit differentiation to show that

$$\frac{dy}{dx} = \frac{2}{3(x+y)^2} - 1$$

[4]

(ii) Hence find the equation of the tangent to the curve

$$(x + y)^3 = 2x$$

at the point (4, –2) in the form $ax + by + c = 0$, where a, b and c are integers. [4]

5 **(a)** Find

$$\int x \cos 2x \, dx$$

[6]

(b) Use the substitution $u = \cot x$ to find

$$\int (1 + \cot^2 x) e^{\cot x} \, dx$$

[7]

6 A function f is defined by

$$f(x) = x^2 - 6x + 5 \qquad x \in R$$

(i) Express f(x) in the form

$$(x - a)^2 - b$$

[3]

(ii) Hence state the range of the function f(x). [1]

(iii) **State** the largest possible domain so that the function f(x) is increasing. [1]

(iv) Find the inverse function $f^{-1}(x)$, for the 1 – 1 function in part **(iii)**, stating its domain. [5]

7 A radioactive isotope decays at a rate proportional to the amount, A, present at time t years.

(i) Form a differential equation to model this. [2]

The half-life of the isotope is 250 years.

(ii) Find the percentage of the original amount that remains after 100 years. [10]

8 **Fig. 1** below shows part of the graph of

$$y = \sin x + \cos x$$

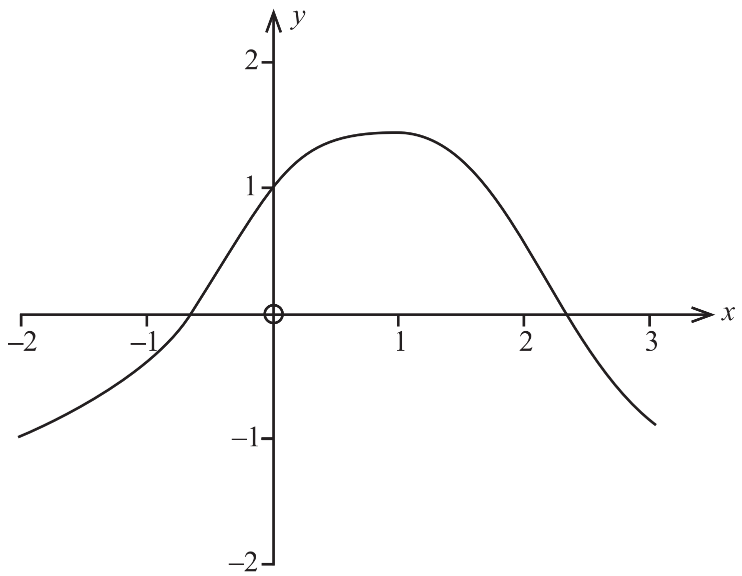

Fig. 1

A glass lampshade is formed by rotating the area enclosed by the curve, the positive x-axis and the y-axis through 2π radians about the x-axis.

Find the **exact** volume enclosed by the lampshade. [10]