Rewarding Learning

ADVANCED
General Certificate of Education
January 2013

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

[AMF21]

AMF21

MONDAY 28 JANUARY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find in terms of π the general solution of the equation

$$1 + \frac{\cos 2\theta}{\cos \theta} = 0$$ [6]

2 **(i)** Show that

$$\sum_{r=1}^{n} r(r+2) = \frac{n}{6}(n+1)(2n+7)$$ [4]

(ii) Using this result, find the sum

$$3 \ln 5 + 4 \ln 5^2 + 5 \ln 5^3 + \ldots + 12 \ln 5^{10}$$

leaving your answer in terms of ln 5 [3]

3 **(i)** Given that

$$\mathrm{f}(x) = \frac{5x^2 + 12x + 14}{(x+2)^2(x^2+1)} \qquad x \neq -2$$

express $\mathrm{f}(x)$ in partial fractions. [8]

(ii) Hence or otherwise show that $\mathrm{f}(x) > 0$ [3]

4 **(a)** Using small angle approximations, show that as $x \to 0$

$$\frac{1 - \cos 4x + x \sin 2x}{x^2} \to 10$$ [3]

(b) Using Maclaurin's theorem, find a series expansion for $\cos 4x$ up to and including the term in x^4 [5]

5 Using the principle of mathematical induction, prove that if n is a positive integer then

$$3^{2n} + 11$$

is divisible by 4 [7]

6 A particle moves so that its displacement x metres from a fixed point O at time t seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + 5\frac{dx}{dt} + 6x = 20\cos t$$

Find the general solution of this equation. [10]

7 **(i)** Show that the equation of the tangent to the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at the point P $(a\cos\theta, b\sin\theta)$ is

$$\frac{x\cos\theta}{a} + \frac{y\sin\theta}{b} = 1$$ [5]

The equation of the tangent to this ellipse at $Q(-a\sin\theta, b\cos\theta)$ is

$$\frac{y\cos\theta}{b} - \frac{x\sin\theta}{a} = 1$$

The tangents at P and Q intersect at the point R.

(ii) Find in terms of θ the coordinates of R. [6]

(iii) Show that as θ varies the locus of R is

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 2$$ [2]

8 **(i)** Using De Moivre's theorem, show that

$$z^n - \frac{1}{z^n} = 2\mathrm{i}\sin n\theta \qquad [3]$$

where $z = \cos\theta + i\sin\theta$

(ii) Using **(i)** with $n = 1$ or otherwise, show that

$$16\sin^5\theta = \sin 5\theta - 5\sin 3\theta + 10\sin\theta \qquad [5]$$

(iii) Hence evaluate

$$\int_0^{\frac{\pi}{3}} \sin^5\theta \, \mathrm{d}\theta \qquad [5]$$

THIS IS THE END OF THE QUESTION PAPER