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ADVANCED
General Certificate of Education
January 2014

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 22 JANUARY, MORNING

AMC41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use the substitution $u = 2x - 3$ to find

$$\int 4x(2x - 3)^5 \, dx \qquad [6]$$

2 **Fig. 1** below shows a vertical tower TO of height $25\,\text{m}$.
The tower is held in place by three cables joined to the top, T, of the tower.
The cables are fastened to the ground at A, B and C.
Taking the tower to lie along the z-axis and the base of the tower, O, to be the origin $(0, 0, 0)$,
the points A, B and T have position vectors:

$$\overrightarrow{OA} = \begin{pmatrix} 2 \\ -6 \\ -1 \end{pmatrix} \quad \overrightarrow{OB} = \begin{pmatrix} 2 \\ 6 \\ 1 \end{pmatrix} \quad \overrightarrow{OT} = \begin{pmatrix} 0 \\ 0 \\ 25 \end{pmatrix}$$

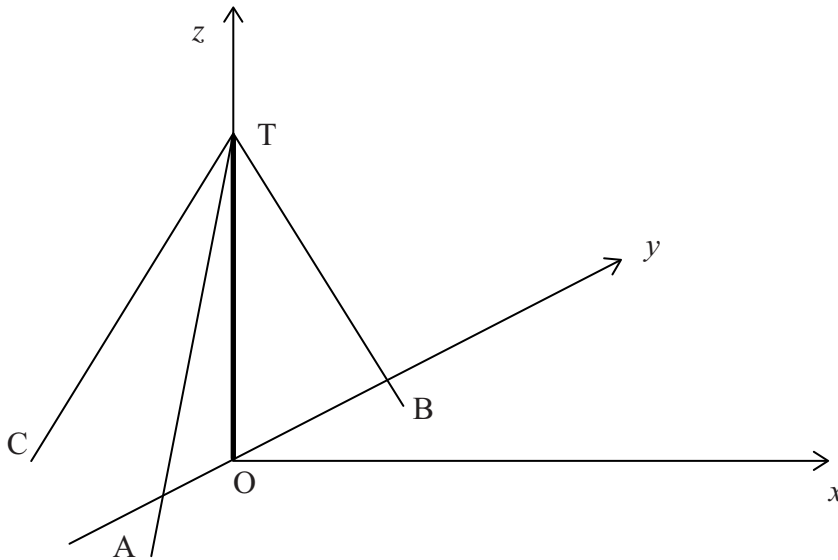**Fig. 1**

(i) Find the lengths of the cables AT and BT. [5]

(ii) Find the angle between the cables AT and BT. [4]

8455   **2**

3   Find the **exact** area bounded by the curve

$$y = \frac{1 - 10x}{(2x + 1)(1 - x)}$$

the x-axis and the lines $x = 2$ and $x = 3$, using partial fractions.   [12]

4   Given that

$$x^2 y - x + 1 = 0$$

use implicit differentiation to show that

$$2y + 4x\frac{dy}{dx} + x^2\frac{d^2 y}{dx^2} = 0$$ [9]

5   The functions f and g are defined as:

$$\text{f}: x \rightarrow \text{e}^x \qquad x \in \mathbb{R} \quad x \geqslant 0$$
$$\text{g}: x \rightarrow 5x + 2 \qquad x \in \mathbb{R}$$

(i)   State the range of f.   [1]

(ii) Find the composite function gf and state its domain and range.   [5]

(iii) Find the **exact** value of x such that $\text{gf}(x) = 11$ [2]

6   Solve the differential equation

$$\frac{dy}{dx} = \frac{\text{cosec } 3y}{\tan x}$$

given that $x = \frac{\pi}{2}$ when $y = \frac{\pi}{3}$ [8]

7 **(i)** Prove the identity

$$\cos 3x \equiv 4 \cos^3 x - 3 \cos x \qquad\qquad [6]$$

(ii) **Hence** solve the equation

$$12 \cos^2 x - \sec x = 9 \qquad\quad 0 \leqslant x \leqslant \pi \qquad\qquad [6]$$

8 A paperweight can be modelled as the solid formed when the area bounded by the curve

$$y = \frac{\sqrt{\ln x}}{x}$$

the x-axis and the line $x = 5$ is rotated through 2π radians about the x-axis.

Find the **exact** volume of the paperweight. [11]

THIS IS THE END OF THE QUESTION PAPER