



ADVANCED  
General Certificate of Education  
January 2014

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## Mathematics

Assessment Unit F2

*assessing*

Module FP2: Further Pure Mathematics 2

[AMF21]

MONDAY 27 JANUARY, MORNING

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### TIME

1 hour 30 minutes.

### INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.  
Answer **all seven** questions.  
Show clearly the full development of your answers.  
Answers should be given to three significant figures unless otherwise stated.  
You are permitted to use a graphic or scientific calculator in this paper.

### INFORMATION FOR CANDIDATES

The total mark for this paper is 75  
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.  
A copy of the **Mathematical Formulae and Tables booklet** is provided.  
Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all seven questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1 (i)** Simplify the expression

$$(\sqrt{r} + \sqrt{r-1})(\sqrt{r} - \sqrt{r-1}) \quad [1]$$

**(ii)** Hence show that  $\sum_{r=1}^n \frac{1}{\sqrt{r} + \sqrt{r-1}} = \sqrt{n}$  [3]

**2** Find, in radians, the general solution of the equation

$$3 \tan^2 \theta = 2 \sin \theta \quad [9]$$

**3 (a)** A root of the equation

$$z^4 - 6z^3 + 23z^2 - 34z + 26 = 0$$

is  $1 + i$

**(i)** State with a reason one other complex root. [2]

**(ii)** Find the remaining roots. [5]

**(b)** Given that  $n$  is a positive integer, show that

$$(1 + i)^{4n} - (1 - i)^{4n} = 0 \quad [5]$$

4 Let  $y = \tan x$

(i) Show that  $\frac{dy}{dx} = 1 + y^2$  and  $\frac{d^2y}{dx^2} = 2y(1 + y^2)$  [3]

(ii) Find, in terms of  $y$ , an expression for  $\frac{d^3y}{dx^3}$  [2]

(iii) Hence or otherwise find the Maclaurin series for  $\tan x$  up to and including the term in  $x^3$  [4]

(iv) Using this approximation for  $\tan x$  and other small angle approximations, find

$$\lim_{x \rightarrow 0} \left( \frac{x \sin^2 x}{\tan x - x \cos x} \right) \quad [3]$$

5 Use mathematical induction to prove that if  $n \geq 1$  and  $\sin x \neq 0$

$$\cos x + \cos 3x + \cos 5x + \dots + \cos(2n-1)x \equiv \frac{\sin 2nx}{2 \sin x} \quad [9]$$

6 The extension  $x$  centimetres of a spiral spring at time  $t$  seconds is given by the differential equation

$$\frac{d^2x}{dt^2} + n^2x = \cos \omega t$$

where  $n$  and  $\omega$  are constants and  $\omega \neq n$

Given that when  $t = 0$ ,  $x = 0$  and  $\frac{dx}{dt} = V$ , express  $x$  as a function of  $t$ . [12]

7 (i) Find the eccentricity of the ellipse

$$\frac{x^2}{9} + \frac{y^2}{4} = 1 \quad [2]$$

(ii) State the coordinates of the foci and the equations of the directrices. [2]

(iii) Show that an equation for the tangent to this ellipse at the point  $P(3 \cos \theta, 2 \sin \theta)$  is

$$\frac{x \cos \theta}{3} + \frac{y \sin \theta}{2} = 1 \quad [4]$$

A line perpendicular to the tangent at P is drawn from the origin and intersects the tangent at Q.

(iv) Find, in terms of  $\theta$ , the coordinates of Q. [5]

(v) Verify that as  $\theta$  varies Q lies on the curve

$$(x^2 + y^2)^2 = 9x^2 + 4y^2 \quad [4]$$

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**THIS IS THE END OF THE QUESTION PAPER**

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