



Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
January 2014

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

FRIDAY 10 JANUARY, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$.

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{A} = \begin{pmatrix} 1 & x & 2 \\ 2 & 3 & 0 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 0 & 6 \\ 1 & x \\ -1 & -3 \end{pmatrix}$

(i) Calculate the matrix product \mathbf{AB} , leaving your answer in terms of x . [2]

(ii) Given that \mathbf{AB} has no inverse, find the value of x . [4]

2 (a) Write down the 2×2 matrix \mathbf{M} which represents a reflection in the line

$$y = \frac{1}{\sqrt{3}}x \quad [3]$$

(b) The matrix $\mathbf{N} = \begin{pmatrix} 1 & 2 \\ 4 & -1 \end{pmatrix}$ represents a linear transformation of the x - y plane.

Find the equations of the straight lines through the origin which are invariant under the transformation given by \mathbf{N} [6]

- 3 (a) The set of numbers $\{1, 3, 7, a\}$, where a is a positive integer, forms a group under multiplication modulo 8

Find the value of a .

[1]

- (b) (i) Copy and complete the following group table for multiplication modulo 18

\times_{18}	1	5	7	11	13	17
1	1	5	7	11	13	17
5	5	7	17	1	11	13
7	7	17	13	5	1	11
11	11	1	5			
13	13	11	1			
17	17	13	11			

[3]

- (ii) Find an element which generates this group, briefly justifying your answer.

[2]

- (iii) Write down a subgroup of this group of order 2

[2]

- (iv) Write down a subgroup of this group of order 3

[2]

- 4 The matrix \mathbf{P} is given by

$$\mathbf{P} = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$$

- (i) Show that the eigenvalues of \mathbf{P} are 2, 3 and 6

[7]

- (ii) For the eigenvalue 6, find a corresponding eigenvector.

[4]

- (iii) Verify that $\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ are eigenvectors of \mathbf{P}

[4]

- (iv) If $\mathbf{U}^T \mathbf{P} \mathbf{U} = \mathbf{D}$, where \mathbf{D} is a diagonal matrix, write down a possible matrix \mathbf{U}

[1]

5 Two circles have equations

$$x^2 + y^2 + 6x - 10y + 9 = 0$$

$$x^2 + y^2 - 2x - 6y + 1 = 0$$

(i) Find the centre and radius of each circle. [4]

(ii) Determine whether or not the circles intersect. Clearly justify your answer. [4]

(iii) Find the point of intersection of the common tangents to the two circles. [5]

6 (a) The complex number z is defined as $z = \frac{3 + i}{1 - i}$

Calculate $z + \frac{1}{z}$, leaving your answer in the form $a + bi$, where a and b are rational numbers. [6]

(b) Given that $(p + qi)^2 = 17 - 6\sqrt{2}i$, find the values of p and the corresponding values of q . [8]

(c) (i) Sketch on an Argand diagram the locus of those points u which satisfy

$$|u - (1 - i)| = 2 \quad [3]$$

(ii) On the same Argand diagram sketch the locus of those points v which satisfy

$$|v - (4 + 5i)| = 1 \quad [1]$$

(iii) Find the maximum value of $|u - v|$ where u and v are the complex numbers which satisfy the equations in (i) and (ii) respectively.

A solution by scale drawing will not be accepted. [3]

THIS IS THE END OF THE QUESTION PAPER
