Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

AMF31

[AMF31]

THURSDAY 26 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **(i)** If $x = 5\cos\theta - 3$ show that

$$16 - 6x - x^2 = 25\sin^2\theta$$ [3]

(ii) Hence or otherwise show that

$$\int \frac{1}{\sqrt{16-6x-x^2}}\,dx = -\cos^{-1}\left(\frac{x+3}{5}\right) + c$$ [4]

2 **(i)** Using the exponential definitions of $\cosh x$ and $\sinh x$ prove that

$$2\cosh 4x\cosh x \equiv \cosh 5x + \cosh 3x$$ [3]

(ii) Hence solve, for real values of x, the equation

$$\cosh 5x + \cosh 3x = 4\cosh x$$

leaving your answers in logarithmic form. [4]

3 **(i)** Sketch the curve $y = \sinh^{-1} x$ [1]

(ii) Show that

$$\frac{d}{dx}(\sinh^{-1} x) = \frac{1}{\sqrt{1+x^2}}$$ [4]

(iii) Show that

$$\sinh^{-1} x \equiv \ln\left(x + \sqrt{1+x^2}\right)$$ [4]

(iv) Find, in fraction form, the exact solution to the equation

$$\sinh^{-1}\frac{3}{4} + \sinh^{-1} x = \sinh^{-1}\frac{4}{3}$$ [4]

4 A plane Π has vector equation

$$\mathbf{r}.(2\mathbf{i} - 3\mathbf{j} + \mathbf{k}) = 5$$

(i) Find the shortest distance from the origin to the plane. [2]

The line L has Cartesian equation

$$\frac{x-2}{3} = \frac{y-5}{-2} = \frac{z+1}{5}$$

(ii) Find the coordinates of the point where the line L meets the plane Π [4]

(iii) Find the angle between the line L and the plane Π [5]

5 **(i)** Given that

$$I_n = \int_0^{\frac{\pi}{2}} x^n \cos x \, dx$$

show that for $n \geqslant 2$

$$I_n = \left(\frac{\pi}{2}\right)^n - n(n-1)\, I_{n-2}$$ [6]

(ii) Hence evaluate, in terms of π,

$$\int_0^{\frac{\pi}{2}} x^4 \cos x \, dx$$ [5]

6 **(a)** **(i)** Show that if

$$y = \sin^{-1}\sqrt{x} - \sqrt{x(1-x)} \qquad |x| < |$$

then $\dfrac{dy}{dx} = \sqrt{\dfrac{x}{1-x}}$ [5]

(ii) Hence or otherwise find the exact value of

$$\int_0^{\frac{1}{2}} \sqrt{\frac{x}{1-x}} \, dx$$ [2]

(b) Show that

$$\int_0^2 \frac{4-3x}{4+3x^2} \, dx = \frac{2\pi}{3\sqrt{3}} - \ln 2$$ [6]

7 With reference to a fixed origin O the points A(4, 1, 3), B(–2, 7, 6) and C(5, –3, 2) determine the plane ABC.

(i) Find $\overrightarrow{AB} \times \overrightarrow{AC}$ [4]

(ii) Hence or otherwise find, in the form $\mathbf{r}.\mathbf{n} = d$, an equation of the plane ABC. [3]

The point D with position vector $\overrightarrow{OD} = 11\mathbf{i} - 9\mathbf{j} + \lambda\mathbf{k}$ is in the plane ABC.

(iii) Find the value of λ. [2]

(iv) What kind of quadrilateral is ABCD? Justify your answer. [2]

(v) Find, in surd form, the area of the quadrilateral ABCD. [2]

THIS IS THE END OF THE QUESTION PAPER