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General Certificate of Education
2011

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

WEDNESDAY 22 JUNE, MORNING

AMM41

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\mathrm{m\,s^{-2}}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

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Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1   A uniform triangular lamina OAB is bounded by the x-axis, the line $x = a$ and the line $y = kx$. O is the origin and A is the point $(a, 0)$. The centre of mass of the lamina is G as shown in **Fig. 1** below.
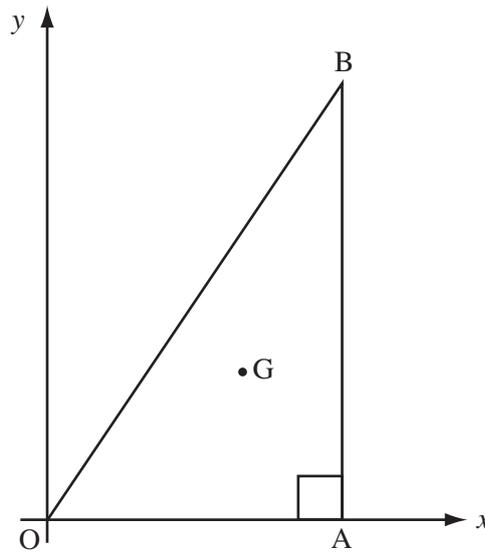

Fig. 1

The mass per unit area of the lamina is σ.

(i)   Find in terms of a, k, and σ, the mass of the lamina and by using integration, its moments about the x- and y-axes.   [8]

(ii) Find the coordinates of G.   [2]

(iii) Verify that for this triangular lamina the coordinates of G can be found by summing the coordinates of the three vertices and dividing by three.   [2]

2   The star-ship FANTASIA has a probon beam engine. The zoomeration, Z, of the star-ship is believed to depend on the powel, P, struk, S and dinesty, D, of the beam.
The dimensions of these quantities are shown in the table below.

Quantity	Dimension
Z	$[L^3][T^{-4}]$
P	$[M][L^{-2}][T]$
S	$[M^2][T^{-2}]$
D	$[M^{-1}][L^3][T^{-1}]$

Boris Blastov conjectures that

$$Z = kP^a S^b D^c$$

where k is a dimensionless constant.

(i)   Use the method of dimensions to find a, b and c.   [8]

During a trial of the engine the following measurements were made in S.I. units:

$$Z = 1.2 \times 10^{11}, P = 2 \times 10^2, S = 3 \times 10^9, D = 5 \times 10^3$$

(ii)   Show that $k = 1.6 \times 10^{12}$ [3]

During further trials, when P was doubled Z increased by a factor of 5

(iii) State whether or not Boris's conjecture could be correct. Briefly explain your answer. [2]

3   Three spheres A, B, and C of equal radius and masses $4m$, $2m$ and m respectively are at rest in a straight line on a smooth horizontal surface. The coefficient of restitution between each pair of spheres is 0.5
A is projected towards B with speed u and collides directly with it.
After the collision A and B move off in the same direction, B with speed w and A with speed v.

(i) Show that $w = u$ and find v.   [6]

A collision now occurs between B and C.

(ii) Find the speeds of B and C after this collision.   [4]

(iii) State if any further collisions will occur, briefly explaining your answer.   [1]

4   In a distant galaxy, the planet Fido has two moons, Ria and Via.
The masses of the planet and its moons are M, m_1 and m_2 respectively.
The orbit of Ria has a radius r and the coplanar orbit of Via has a radius $2.08r$
The universal gravitational constant is G.
Ria moves around its orbit at speed v_1 and Via moves at v_2

(i) Show that $v_1 = 1.44v_2$ approximately.   [5]

The angular velocities of Ria and Via are ω_1 and ω_2 respectively.

(ii) By using the result in **(i)**, or otherwise, show that $\omega_1 = 3\omega_2$ approximately.   [3]

The period of Via is 63 days.

(iii) Find the period of Ria.   [3]

5 The "Dip" is part of a competitive cycle trail where competitors must keep their bikes in contact with the track at all times.
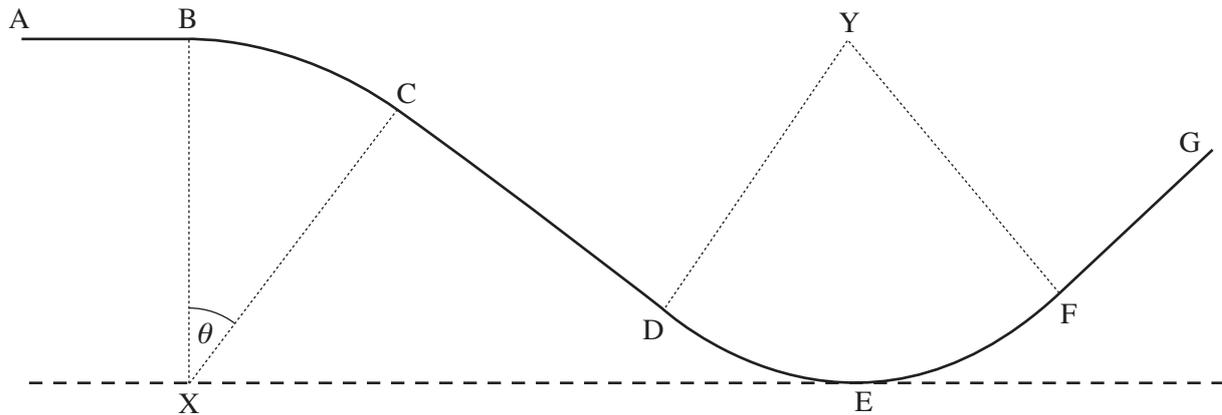**Fig. 2** below shows the vertical section along the "Dip".

Fig. 2

There is a horizontal approach AB, a circular arc BC, centre X, a straight incline CD and another circular arc DEF, centre Y, followed by an upward incline FG.
Both arcs are of radius 5 m. BC subtends $\theta°$ at its centre X where $\cos\theta = 0.8$
A, B and Y are at the same horizontal level.
E is 5 m vertically below the level of A and B.
Decla approaches the "Dip" along AB reaching B with a speed of $4\,\text{m s}^{-1}$
Model Decla and her bike as a particle of mass 70 kg.

(i) Find the force exerted by the track on the bike and rider at E. [7]

At C, Decla and her bike experience a normal reaction, *N* newtons, from the track.

(ii) Find *N*. [6]

(iii) Explain briefly why the bike remains in contact with the track over the arc BC. [2]

(iv) State what further modelling assumption you have made in answering this question. [1]

6 The triangle ABC has sides:

$$AB = 0.6\,m$$
$$BC = 0.8\,m$$
$$AC = 1.0\,m$$

D is the point on AC such that BD is perpendicular to AC.
Forces of 12 N, 9 N and 8 N act along the sides BA, BC and AC respectively as shown in **Fig. 3** below.
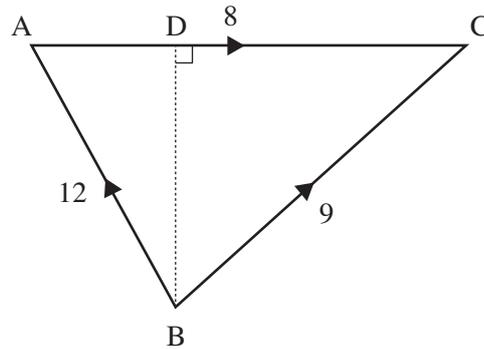

Fig. 3

The resultant of these forces is *R*.

(i) Show that *R* passes through D and find its magnitude. [7]

The forces are now replaced and 3 N acts along BA, 5 N along AC and 4 N along CB as shown in **Fig. 4** below.
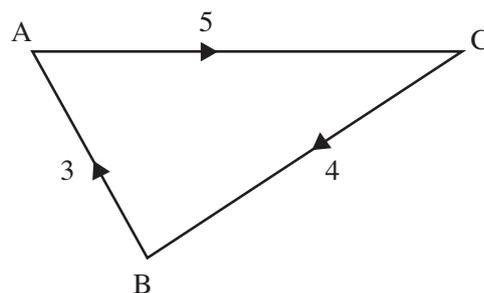

Fig. 4

(ii) Show that this system reduces to a couple and find its moment. [5]

THIS IS THE END OF THE QUESTION PAPER

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