



ADVANCED
General Certificate of Education
2011

Mathematics
Assessment Unit M4
assessing
Module M4: Mechanics 4
[AMM41]

WEDNESDAY 22 JUNE, MORNING

MARK
SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

1	(i) $M = \frac{1}{2}a \times ka \times \sigma = \frac{\sigma ka^2}{2}$	MW1	AVAILABLE MARKS
	$M_{\sigma_y} = \int_0^a \sigma xy dx$	$M_{\sigma_x} = \frac{1}{2} \int_0^a \sigma y^2 dx$	M1 M1
			MW1 MW1
	$= \int_0^a \sigma kx^2 dx$	$= \frac{1}{2} \int_0^a \sigma k^2 x^2 dx$	
	$= \left[\frac{\sigma kx^3}{3} \right]_0^a$	$= \frac{1}{2} \left[\frac{\sigma k^2 x^3}{3} \right]_0^a$	MW1 MW1
	$= \frac{\sigma ka^3}{3}$	$= \frac{\sigma k^2 a^3}{6}$	MW1
	(ii) $\bar{x} = \frac{\sigma ka^3}{3} \times \frac{2}{\sigma ka^2} = \frac{2}{3}a$		MW1
	$\bar{y} = \frac{\sigma k^2 a^3}{6} \times \frac{2}{\sigma ka^2} = \frac{1}{3}ka$		MW1
	(iii) coordinates (a, ka) $(a, 0)$ $(0, 0)$		MW1
	sum and divide by 3 $\left(\frac{2a}{3}, \frac{ka}{3} \right)$	W1	12

2 (i) $Z = kP^a S^b D^c$

$$[L^3][T^{-4}] = [M^a][L^{-2a}][T^a][M^{2b}][T^{-2b}][M^{-c}][L^{3c}][T^{-c}]$$

$$= [M^{a+2b-c}][L^{-2a+3c}][T^{a-2b-c}]$$

Equate indices

$$[M] \quad 0 = a + 2b - c \quad \text{①} \quad \text{MW1}$$

$$[L] \quad 3 = -2a + 3c \quad \text{②} \quad \text{MW1}$$

$$[T] \quad -4 = a - 2b - c \quad \text{③} \quad \text{MW1}$$

$$\text{①} + \text{③} \quad -4 = 2a - 2c \quad \text{④}$$

$$\text{②} + \text{④} \quad -1 = c \quad \text{M1}$$

$$\text{sub into ②} \quad 3 = -2a - 3$$

$$a = -3 \quad \text{W1}$$

$$\text{sub into ①} \quad 0 = -3 + 2b + 1$$

$$b = 1 \quad \text{W1}$$

(ii) $Z = \frac{kS}{P^3 D}$

$$\text{so } k = \frac{P^3 D Z}{S} \quad \text{MW1}$$

$$= \frac{1.2 \times 10^{11} \times 8 \times 10^6 \times 5 \times 10^3}{3 \times 10^9} \quad \text{MW1}$$

$$= 1.6 \times 10^{12} \quad \text{W1}$$

(iii) Doubling would decrease Z by a factor of 8 W1

So, Boris's conjecture is wrong MW1

AVAILABLE
MARKS

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3 (i)

$$\begin{array}{c} \vec{u} \\ \textcircled{4m} \\ \vec{v} \end{array}$$

$$\begin{array}{c} 0 \\ \vec{0} \\ \textcircled{2m} \\ \vec{w} \end{array}$$

Conservation of Momentum

$$2mw + 4mv = 4mu$$

M1 W1

 \Rightarrow

$$w + 2v = 2u$$

Newton's Law

$$w - v = -\frac{1}{2}(0 - u)$$

M1

$$= \frac{1}{2}u$$

W1

$$3v = \frac{3}{2}u$$

$$v = \frac{1}{2}u$$

MW1

$$\therefore w = u$$

MW1

(ii)

$$\begin{array}{c} \vec{u} \\ \textcircled{2m} \\ \vec{p} \end{array}$$

$$\begin{array}{c} 0 \\ \vec{0} \\ \textcircled{m} \\ \vec{q} \end{array}$$

Conservation of Momentum

$$mq + 2mp = 2mu$$

MW1

Newton's Law

$$q - p = \frac{1}{2}u$$

MW1

same equations

M1

$$\therefore p = \frac{1}{2}u, \quad q = u$$

W1

(iii) Situation is

$$\begin{array}{c} \textcircled{4m} \\ \vec{\frac{1}{2}u} \end{array}$$

$$\begin{array}{c} \textcircled{2m} \\ \vec{\frac{1}{2}u} \end{array}$$

$$\begin{array}{c} \textcircled{m} \\ \vec{u} \end{array}$$

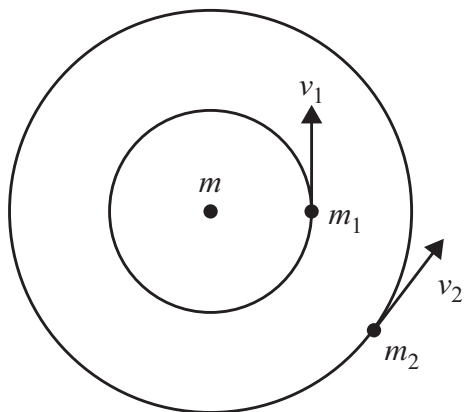
 \therefore no more collisions

MW1

11

AVAILABLE
MARKS

4 (i)



$$\frac{GMm_1}{r^2} = \frac{m_1 v_1^2}{r}$$

$$\frac{GMm_2}{2.08^2 r^2} = \frac{m_2 v_2^2}{2.08r}$$

$$\frac{v_1^2}{v_2^2} = 2.08$$

M1 W1

MW1

MW1

$$v_1 = \sqrt{2.08} v_2$$

$$v_1 = 1.44 v_2$$

$$(ii) \quad v_1 = r \omega_1 \quad v_2 = 2.08 r \omega_2$$

$$\text{sub in} \quad r \omega_1 = 1.44 \times 2.08 r \omega_2$$

$$\omega_1 = 3 \omega_2$$

W1

M1

W1

W1

$$(iii) \quad \frac{P_1}{P_2} = \frac{\frac{2\pi}{\omega_1}}{\frac{2\pi}{\omega_2}} = \frac{\omega_2}{\omega_1} = \frac{1}{3}$$

$$\therefore P_1 = \frac{1}{3} P_2 = 21 \text{ days}$$

M1 W1

W1

AVAILABLE
MARKS

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5 (i) By Conservation of Energy

$$\frac{1}{2} \times 70 \times 16 + 70 \times 9.8 \times 5 = \frac{1}{2} \times 70 \times v^2$$

$$\therefore v^2 = 16 + 98 = 114$$

$$ma = m \frac{v^2}{r}$$

$$\therefore \frac{70v^2}{5} = R - 70g$$

$$R = 70(9.8 + \frac{114}{5})$$

$$= 2282 \text{ N}$$



M1

W1

W1

M1

M1 W1

W1

(ii)

By Conservation of Energy

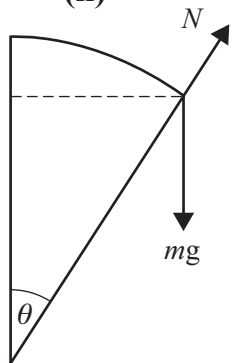
$$\frac{1}{2} \times 70 \times 16 + 70 \times 9.8 \times 5(1 - 0.8) = \frac{1}{2} \times 70 \times v^2$$

$$v^2 = 16 + 19.6 = 35.6$$

$$R - \frac{70v^2}{5} = 70g \times 0.8 - N$$

$$N = 70(7.84 - 7.12)$$

$$= 50.4 \text{ N}$$



M1 W1

W1

M1 W1

W1

(iii) $N = 70(g \cos \theta - \frac{v^2}{5})$ on BC

$\cos \theta$ is decreasing and v^2 increasing along BC so N_{min} will occur at C and is > 0

so, bike and rider stay in contact with the track

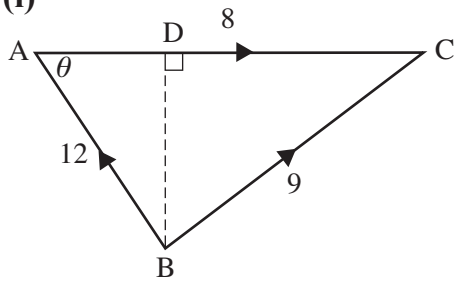
M1 W1

(iv) Any forces other than mg that act on the bike and rider do no work.

MW1

16

6 (i)



$$\sin \theta = 0.8$$

$$\cos \theta = 0.6$$

Resolving for the 12 N and 9 N

$$\uparrow 12 \times 0.8 + 9 \times 0.6 = 15$$

$$\leftrightarrow 9 \times 0.8 - 12 \times 0.6 = 0$$

MW1

M1

W1

W1

The resultant of these forces acts vertically through B i.e. along BD.

The 8 N acts through D so the overall resultant will too.

M1

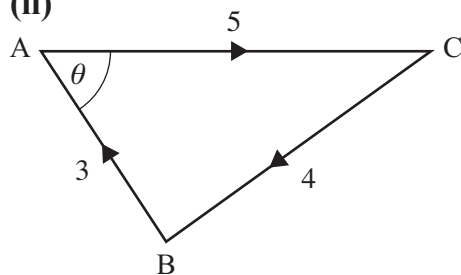
Pythagorean triangle 8, 15, R

M1

So $R = 17$

W1

(ii)



$$\updownarrow 3 \times 0.8 - 4 \times 0.6 = 0$$

$$\leftrightarrow 5 - 3 \times 0.6 - 4 \times 0.8 = 0$$

$$\curvearrowright 4 \times 0.6 = 2.4$$

M1

W1

M1 W1

A zero resultant force and a non-zero moment \Rightarrow a couple and its moment is 2.4 Nm clockwise.

M1

TotalAVAILABLE
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12

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