Rewarding Learning

ADVANCED
General Certificate of Education
2011

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

WEDNESDAY 1 JUNE, MORNING

MARK SCHEME

		AVAILABLE MARKS
1 **(i)** $\overrightarrow{AB} = \overrightarrow{AO} + \overrightarrow{OB} = -3\mathbf{i} + \mathbf{j} + 2\mathbf{i} + 6\mathbf{j}$		
$= -\mathbf{i} + 7\mathbf{j}$	M1W1	
(ii) $\lvert\overrightarrow{AB}\rvert = \sqrt{(-1)^2 + (7)^2} = \sqrt{50} = 5\sqrt{2}$	M1W1	
(iii) $\overrightarrow{OA} \cdot \overrightarrow{OB} = (3\mathbf{i} - \mathbf{j}) \cdot (2\mathbf{i} + 6\mathbf{j})$		
$= 6 - 6$	M1	
$= 0$	W1	
(iv) 90°	MW1	7
2 **(i)** $x + xy - 12$		
$1 + y + \frac{dy}{dx} x$	M2W2	
(ii) at (2, 5)		
$1 + 5 + 2\frac{dy}{dx} = 0$		
$\frac{dy}{dx} = -3$	MW1	
Eqn: tangent $y - 5 = -3(x - 2)$	M1	
$y = -3x + 11$	W1	
$y + 3x = 11$		7

				AVAILABLE MARKS
3	**(i)**	$2\cos x + 4\sin x = R\cos(x-\alpha)$		
		$\cos\alpha = 2/R$		
		$\sin\alpha = 4/R$		
		$\tan\alpha = 2 \Rightarrow \alpha = 63.43°$	M1W1	
		$= 63.4°$ (3.s.f.)		
		$R = \sqrt{2^2+4^2} = \sqrt{20} = 2\sqrt{5}$	M1W1	
		$2\sqrt{5}\cos(x-63.4°)$		
	(ii)	$2\cos x + 4\sin x = 3$		
		$\sqrt{20}\cos(x-63.4°) = 3$	M1	
		$\cos(x-63.4°) = \dfrac{3}{\sqrt{20}}$		
		$x - 63.4° = 47.8695°$ or $312.13°$	MW2	
		$x = 111°$ or $15.6°$	W2	9
4	**(i)**	Volume $= \int_{-12}^{3} \pi y^2 \, dx$	M2	
		$= \int_{-12}^{3} \pi(144 - x^2)\, dx$	MW1	
		$= \pi\left[144x - \dfrac{x^3}{3}\right]_{-12}^{3}$	MW1	
		$= \pi\left[(432-9) - (-1728+576)\right]$	M1	
		$= 1575\,\pi\ \text{cm}^3$	W2	
	(ii)	The bowl needs a flat bottom	MW1	8

		AVAILABLE MARKS

5 **(i)** $\sin(A+B) = \sin A \cos B + \cos A \sin B$ M1

$\sin(A+A) = \sin A \cos A + \cos A \sin A$ M1

$= 2 \sin A \cos A$ W1

(ii) LHS $\tan A + \cot A$

$\dfrac{\sin A}{\cos A} + \dfrac{\cos A}{\sin A}$ M1W1

$\dfrac{\sin^2 A + \cos^2 A}{\sin A \cos A}$ MW1

$\dfrac{1}{\frac{1}{2}\sin 2A}$ M2

$\dfrac{2}{\sin 2A} = \text{R.H.S.}$ W1 9

6 $\dfrac{dx}{dt} = k(3-x)(4-x)$

$\displaystyle\int_0^2 \frac{dx}{(3-x)(4-x)} = \int_0^{10} k\,dt$ M2W2

$\dfrac{1}{(3-x)(4-x)} = \dfrac{A}{3-x} + \dfrac{B}{4-x}$ MW1

$1 = A(4-x) + B(3-x)$ MW1

Let $x = 4$ $\quad 1 = -B \Rightarrow B = -1$ MW2

$x = 3$ $\quad 1 = A$

$\dfrac{1}{3-x} - \dfrac{1}{4-x}$

$\displaystyle\int_0^2 \frac{1}{3-x} - \frac{1}{4-x}\,dx = \int_0^{10} k\,dt$ MW1

			AVAILABLE MARKS
	$\left[-\ln \mid 3-x \mid + \ln \mid 4-x \mid \right]_0^2 = \left[k\,t \right]_0^{10}$	W3	
	$\left[\ln \mid \frac{4-x}{3-x} \mid \right]_0^2 = 10\,k$		
	$\ln 2 - \ln \frac{4}{3} = 10\,k$		
	$k = \frac{1}{10} \ln \frac{3}{2}$	W2	
	$[\, k = 0.0405 \,]$		14
7	**(i)** $x = 1 \qquad y = 2$	MW2	
	(ii) $\text{f} : x \rightarrow \frac{2x+3}{x-1}$		
	Let $y = \frac{2x+3}{x-1}$	M1	
	$y\,(x-1) = 2x + 3$	M1	
	$yx - 2x = y + 3$		
	$x = \frac{y+3}{y-2}$	M1W1	
	$\text{f}^{-1} : x \rightarrow \frac{x+3}{x-2}$	W1	
	domain $x > 2$	MW1	8

	AVAILABLE MARKS

8 **(i)** $\int x \quad \ln x \quad dx$

int diff — M1

$= \frac{x^2}{2} \ln x - \int \frac{x^2}{2} \frac{1}{x} dx$ — W3

$= \frac{x^2}{2} \ln x - \frac{x^2}{4} + c$ — W2

(ii) **(a)** $\int x \ln x^2 \, dx$

$= \int 2x \ln x \, dx$ — M1W1

$= x^2 \ln x - \frac{x^2}{2} + d$ — MW1

(b) $\int x \ln 3x \, dx$

$= \int x (\ln 3 + \ln x) \, dx$ — M1W1

$= \int x \ln 3 + x \ln x \, dx$

$= \frac{x^2}{2} \ln 3 + \frac{x^2}{2} \ln x - \frac{x^2}{4} + g$ — MW2 — 13

Total — **75**