Rewarding Learning

ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

MONDAY 25 JUNE, AFTERNOON

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

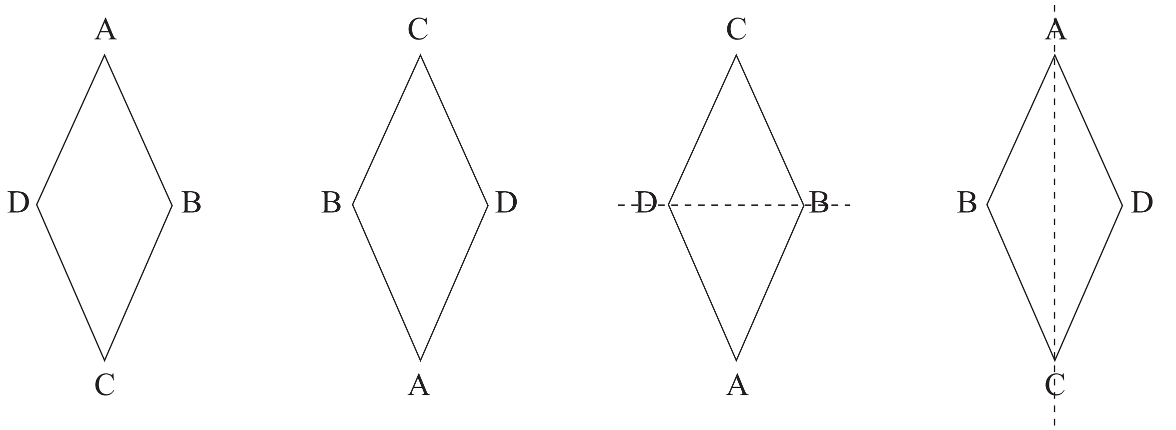
(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

				AVAILABLE MARKS
1	(i)	$\begin{vmatrix} \lambda & 5 \\ 4 & 5\lambda \end{vmatrix} = 0$ if no unique solution exists	M1W1	
		$\Rightarrow 5\lambda^2 - 20 = 0$	M1W1	
		$\Rightarrow \lambda^2 = 4$		
		$\Rightarrow \lambda = \pm 2$	W2	
	(ii)	If $\lambda = 2$, the equations become		
		$2x + 5y = 11$		
		$4x + 10y = \mu$	M1	
		For infinite solutions, $\mu = 2 \times 11 = 22$	W1	8
2	(i)	det $(\mathbf{M} - \lambda\mathbf{I}) = 0$	M1	
		$\Rightarrow \begin{vmatrix} 2-\lambda & 1 & -1 \\ 1 & 3-\lambda & 1 \\ 1 & 0 & 4-\lambda \end{vmatrix} = 0$	W1	
		$\Rightarrow (2 - \lambda)\,[(3 - \lambda)\,(4 - \lambda) - 0] - 1[4 - \lambda - 1] - 1[0 - 3 + \lambda] = 0$	M1W1	
		$\Rightarrow (2 - \lambda)\,(3 - \lambda)\,(4 - \lambda) - 3 + \lambda + 3 - \lambda = 0$		
		$\Rightarrow (2 - \lambda)\,(3 - \lambda)\,(4 - \lambda) = 0$	W1	
		$\Rightarrow \lambda = 2, 3, 4$	W2	
	(ii)	For $\lambda = 2$, $\begin{pmatrix} 2 & 1 & -1 \\ 1 & 3 & 1 \\ 1 & 0 & 4 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 2\begin{pmatrix} x \\ y \\ z \end{pmatrix}$	M1	
		$\Rightarrow 2x + y - z = 2x \quad \Rightarrow y = z$		
		$x + 3y + z = 2y \quad \Rightarrow x = -2z$		
		$x + 4z = 2z \quad \Rightarrow x = -2z$	M1	
		$\Rightarrow x = -2z,\ y = z$	W1	
		Therefore, a possible eigenvector is $\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	W1	
		The corresponding unit eigenvector is $\frac{1}{\sqrt{6}}\begin{pmatrix} -2 \\ 1 \\ 1 \end{pmatrix}$	MW1	12

AVAILABLE MARKS

3 (i)



I – no change

R – rotation of 180° about centre

S – reflection in DB

T – reflection in AC

MW4

(ii)

	I	R	S	T
I	I	R	S	T
R	R	I	T	S
S	S	T	I	R
T	T	S	R	I

MW4

(iii)

	1	4	11	14
1	1	4	11	14
4	4	1	14	11
11	11	14	1	4
14	14	11	4	1

MW4

(iv) The groups are isomorphic. MW1

In each group all the elements are self inverse and a possible isomorphism is I ↔ 1, R ↔ 4, S ↔ 11, T ↔ 14 MW1 14

			AVAILABLE MARKS
4	**(a)** $\cos\theta = \frac{\sqrt{3}}{2}$ and $\sin\theta = \frac{1}{2}$		
	Hence $\theta = 30°$	MW1	
	Therefore the matrix represents a rotation of 30° anticlockwise about the origin	MW3	
	(b) $\begin{pmatrix}1 & 2\\4 & -1\end{pmatrix}\begin{pmatrix}x\\3x-2\end{pmatrix} = \begin{pmatrix}X\\Y\end{pmatrix}$	M1	
	$\Rightarrow x + 6x - 4 = X$		
	and $4x - 3x + 2 = Y$	MW1	
	$\Rightarrow X = 7x - 4$		
	and $Y = x + 2$		
	$\Rightarrow x = Y - 2$	M1	
	Substitute to give $X = 7(Y - 2) - 4$		
	Therefore the transformed line has the equation $X = 7Y - 18$	W1	8
5	**(i)** $x^2 + y^2 + 2x - 4 = 0$ ①		
	$x^2 + y^2 + 8x + 2y - 8 = 0$ ②		
	Subtract to give $6x + 2y - 4 = 0$	M1	
	$\Rightarrow y = 2 - 3x$ ③	W1	
	Substitute ③ into ①	M1	
	$\Rightarrow x^2 + (2 - 3x)^2 + 2x - 4 = 0$	W1	
	$\Rightarrow x^2 + 4 - 12x + 9x^2 + 2x - 4 = 0$	W1	
	$\Rightarrow 10x^2 - 10x = 0$		
	$\Rightarrow 10x(x - 1) = 0$	MW1	
	$\Rightarrow x = 0, 1$	W1	
	Using ③, $y = 2, -1$		
	Therefore the points of intersection are (0, 2) and (1, –1)	W1	
	(ii) Substitute $y = 2x + k$ into the equation of C_1	M1	
	$\Rightarrow x^2 + (2x + k)^2 + 2x - 4 = 0$	W1	
	$\Rightarrow x^2 + 4x^2 + 4kx + k^2 + 2x - 4 = 0$		
	$\Rightarrow 5x^2 + 2x(2k + 1) + (k^2 - 4) = 0$	W1	
	If line is a tangent, then the equation has equal roots i.e. $b^2 - 4ac = 0$	M1	
	$\Rightarrow 4(2k + 1)^2 - 4(5)(k^2 - 4) = 0$		
	$\Rightarrow (2k + 1)^2 - (5)(k^2 - 4) = 0$		
	$\Rightarrow 4k^2 + 4k + 1 - 5k^2 + 20 = 0$		
	$\Rightarrow -k^2 + 4k + 21 = 0$		
	$\Rightarrow k^2 - 4k - 21 = 0$	W1	
	$\Rightarrow (k + 3)(k - 7) = 0$		
	$\Rightarrow k = -3, 7$	W1	14

AVAILABLE MARKS

6 **(a)** $z_1 + 2z_2 = (3 + 2) + (4 + 2p)\mathrm{i}$

$= 5 + (4 + 2p)\mathrm{i}$ MW1

If this is a real number, then $4 + 2p = 0$ M1

Hence $p = -2$ W1

(b) $\frac{5 - 2\mathrm{i}}{3 + \mathrm{i}} \times \frac{3 - \mathrm{i}}{3 - \mathrm{i}}$ M1W1

$= \frac{15 - 6\mathrm{i} - 5\mathrm{i} - 2}{9 + 1}$ W2

$= \frac{13 - 11\mathrm{i}}{10}$

(c) **(i)** The perpendicular bisector of the line joining the points (3, 0) and (7, 2) – see Line (i) in **Fig. 1** below. MW3

(ii) The half-line through the point (3, 2) at an angle $\frac{\pi}{4}$ to the horizontal – see Line (ii) in **Fig. 1** below. MW3

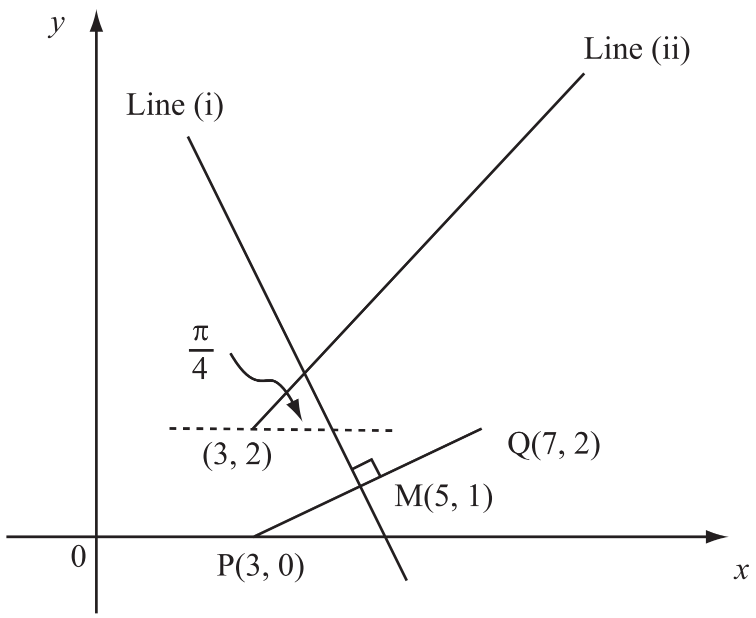


Fig. 1

(iii) Gradient of PQ $= \frac{2 - 0}{7 - 3} = \frac{2}{4} = \frac{1}{2}$ MW1

Hence gradient of L_1 is -2 MW1

The equation of L_1 is then given by $y - 1 = -2(x - 5)$

$\Rightarrow y = -2x + 11$ MW1

The equation of L_2 is given by $y - 2 = 1(x - 3)$

$\Rightarrow y = x - 1$ MW1

Solving simultaneously, gives

$\Rightarrow -2x + 11 = x - 1$ M1

$\Rightarrow 3x = 12$

$\Rightarrow x = 4$

$\Rightarrow y = 3$

Therefore, the point of intersection of the loci is (4, 3) W1 19

Total **75**