Rewarding Learning

ADVANCED
General Certificate of Education
2012

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

FRIDAY 22 JUNE, AFTERNOON

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

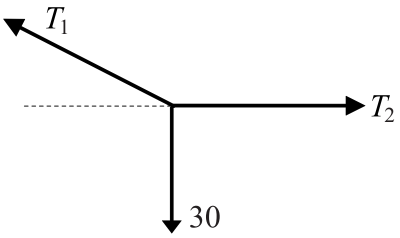
AVAILABLE MARKS

1 **(a)** **(i)** M (B with a curved arrow above it) $0.3P = 0.5 \times 30$ M1 M1

$P = \frac{5}{3} \times 30 = 50\,\text{N}$ W1

(ii) at A, P and the tension in AC are in line and AB is $\perp_R$ to their line of action $\therefore$ no force is induced in AB as no other force acts at A M1

(iii) at C



R $\updownarrow$ $T_1 \sin \alpha = 30$ M1

$T_1 = \frac{5 \times 30}{3} = 50\,\text{N}$ W1

Alternative R $\nwarrow\!\searrow$ $P = T_1 = 50$

R $\leftrightarrow$ $T_2 = T_1 \cos \alpha$

$= \frac{4}{5} T_1 = 40\,\text{N}$ MW1

(b) Trig Values MW1

R $\longleftrightarrow$ $9 \times 0.6 + 12 \times 0.8 - 17 \times \frac{15}{17} = 5.4 + 9.6 - 15 = 0$ M1W1

R $\updownarrow$ $9 \times 0.8 - 12 \times 0.6 - 8 + 17 \times \frac{8}{17} = 7.2 - 7.2 - 8 + 8 = 0$ MW1

M (A with a curved arrow above it) $12 \times 0.9 + 8 \times 1.5 = 10.8 + 12 = 22.8$ Nm M1W1

zero/No Resultant force and non zero moment $\Rightarrow$ couple M1 14

		AVAILABLE MARKS
2 **(i)** $W = cAdv^2$		
l.h.s. $[W] = [M][L][T^{-2}]$	MW1	
r.h.s $= [L^2][M][L^{-3}][L^2][T^{-2}]$	M1	
$= [M][L][T^{-2}]$	W1	
(ii) $P = \frac{W}{t} = \frac{F \times s}{t}$	M1	
$[P] = [M][L][T^{-2}][L][T^{-1}]$		
$= [M][L^2][T^{-3}]$	W1	
(iii) $[M][L^2][T^{-3}] = [L^{2x}][M^y][L^{-3y}][L^z][T^{-z}]$		
$= [M^y][L^{2x-3y+z}][T^{-z}]$	MW1	
equating indices	M1	
[M] $y = 1$	W1	
[L] $2x - 3y + z = 2$	W1	
[T] $-z = -3 \quad \therefore \quad z = 3$	W1	
$\therefore 2x - 3y + 3 = 2$		
and $x = 1$	MW1	
$\therefore P = k\,Adv^3$		
(iv) $\frac{P}{W} = \frac{kAdv^3}{cAdv^2} = \frac{kv}{c}$	MW1	
$\therefore$ power to weight ratio $\propto v$		12

AVAILABLE MARKS

3 **(i)**

$\rightarrow u$ $\quad \rightarrow 0$

$(2m)$ $\quad (m)$

$\rightarrow v_1$ $\quad \rightarrow v_2$

Newton $\quad v_2 - v_1 = -\frac{1}{2}(0 - u) = \frac{1}{2}u$ — M1W1

Cons. Mom. $\cancel{m}v_2 + 2\cancel{m}v_1 = 0 + 2\cancel{m}u$ — M1W1

$$3v_1 = \frac{3}{2}u$$

$$v_1 = \frac{1}{2}u$$ — MW1

$$\therefore v_2 = u$$ — MW1

(ii)

$\rightarrow u$ $\quad \rightarrow 0$

(m) $\quad (km)$

$\rightarrow w_1$ $\quad \rightarrow w_2$

Newton $\quad w_2 - w_1 = \frac{1}{2}u$ — MW1

Cons. Mom. $k\cancel{m}w_2 + \cancel{m}w_1 = \cancel{m}u$ — MW1

$$(1 + k)w_2 = \frac{3}{2}u$$

$$w_2 = \frac{3u}{2(1 + k)}$$ — MW1

and $\quad w_1 = w_2 - \frac{1}{2}u$ — MW1

(iii)

$$\frac{3u}{2(1 + k)} - \frac{1}{2}u < \frac{1}{2}u$$ — M1

$$\frac{3}{2(1 + k)} < 1$$

$$3 < 2(1 + k)$$

$$1 < 2k$$

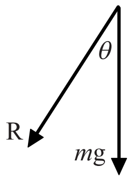
$$k > \frac{1}{2}$$ — W1 — 12

AVAILABLE MARKS

4 **(i)** $\theta = \pi \quad \therefore \frac{1}{2}mv^2 = mgr(1-(-1)) = 2mgr$ MW1

$v = 2\sqrt{gr}$ W1

(ii)



Re ↙ $R + mg\cos\theta = \frac{mv^2}{r}$ M1M1W1

$= 2mg(1 - \cos\theta)$

$R = mg(2 - 3\cos\theta)$ MW1

(iii) $R = mg(2 - 3\cos\theta) = 0$ M1

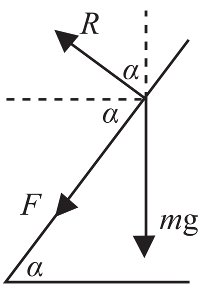
$\cos\theta = \frac{2}{3}$

$\theta = 48.189°$

$48.2°$ W1

(iv)
- K.E. just = 0 at A if A can be reached MW1
- In **(ii)** R can be negative and thrust out as well as in so bead can complete the circle M1
- In **(iii)** the string cannot thrust out, it goes slack and so circular motion ceases – it's projectile motion then M1 — 11

5 **(i)**



$\cos\alpha = \sin\alpha = \frac{1}{\sqrt{2}}$ MW1

Re ← $\frac{mv^2}{50} = F\cos\alpha + R\sin\alpha$ M1M1W1

Re ↕ $mg = R\cos\alpha - F\sin\alpha$ MW1

$F = \mu R$ MW1

$\therefore \frac{v^2}{490} = \frac{\mu R\cos\alpha + R\sin\alpha}{R\cos\alpha - \mu R\sin\alpha}$ M1

$= \frac{1+\mu}{1-\mu}$ as $\alpha = 45°$

$v^2 = 490\left(\frac{1+\mu}{1-\mu}\right)$ W1

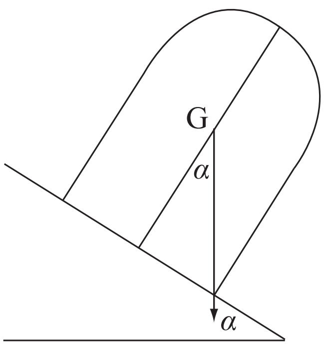
(ii) Since $F \to -F$ then we replace μ by $-\mu$ M1

so $u^2 = 490\left(\frac{1-\mu}{1+\mu}\right)$ W1

(iii) $\frac{v^2}{u^2} = \left(\frac{1+\mu}{1-\mu}\right)^2$ M1

$\frac{v}{u} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$

$\therefore v = 3u$ W1 — 12

				AVAILABLE MARKS
6	(i)	$M_y = \pi\rho \int_0^2 xy^2 \, dx$	M1	
		$y = 0, \quad x = 2 \quad \text{as} \quad x > 0$	MW1	
		$\therefore M_y = \pi\rho \int_0^2 x\left(1 - \frac{x^2}{4}\right) dx$	W1	
		$= -2\pi\rho \int_0^2 \left(-\frac{x}{2}\right)\left(1 - \frac{x^2}{4}\right) dx \quad \text{or} \quad \pi\rho \int_0^2 \left(x - \frac{x^3}{4}\right) dx$		
		$= -2\pi\rho \frac{1}{2}\left[\left(1 - \frac{x^2}{4}\right)^2\right]_0^2 \qquad = \pi\rho \left[\frac{x^2}{2} - \frac{x^4}{16}\right]_0^2$	MW1	
		$= -\pi\rho(0 - 1) \qquad = \pi\rho\big((2 - 1) - (0)\big)$		
		$= \pi\rho \qquad = \pi\rho$	W1	
	(ii)	$m = \pi\rho \int_0^2 y^2 \, dx$		
		$= \pi\rho \int_0^2 \left(1 - \frac{x^2}{4}\right) dx$	MW1	
		$= \pi\rho \left[x - \frac{x^3}{12}\right]_0^2$	MW1	
		$= \pi\rho\left(\left(2 - \frac{8}{12}\right) - (0)\right)$		
		$= \frac{4}{3}\pi\rho$	W1	
		$\therefore \bar{x} = \frac{M_y}{m} = \frac{3}{4}$	MW1	
	(iii)	 Dia.	M1	
		$x = 0, \quad y = 1$	MW1	
		$\tan\alpha = \frac{1}{\frac{3}{4}} = \frac{4}{3}$	M1 W1	
		$\alpha = \tan^{-1}\frac{4}{3} = 53.1°$	W1	14
			Total	**75**