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ADVANCED
General Certificate of Education
2013

Mathematics

Assessment Unit C4

assessing

Module C4: Core Mathematics 4

[AMC41]

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THURSDAY 6 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 The motion of a toy car on a race track can be modelled by the equations

$$x = 3\cos t \quad \text{and} \quad y = \sin 2t$$

Find $\frac{dy}{dx}$ when $t = 2$ [5]

2 **a** and **b** are two vectors where

$$\mathbf{a} = t\mathbf{i} + 5\mathbf{j} + \mathbf{k}$$

$$\mathbf{b} = 2t\mathbf{i} + t\mathbf{j} + 2\mathbf{k}$$

(i) State the value of t for which the vectors are parallel. [1]

(ii) Find the values of t for which the vectors are perpendicular. [6]

3 **(a)** **(i)** Use partial fractions to find A and B where

$$\frac{4}{x(2-x)} = \frac{A}{x} + \frac{B}{2-x}$$ [4]

(ii) Hence find

$$\int \frac{4}{2x - x^2}\, dx$$ [3]

(b) Use integration by parts to find

$$\int x\, e^{3x}\, dx$$ [5]

4 The area under the curve

$$y = 2\sqrt{x} + 1$$

between $x = 0$ and $x = 4$ is rotated through 360° about the x-axis.

Find the volume of the body so formed. [7]

5 **(a)** Sketch the graph of $y = \cos^{-1} x$, stating its domain. [3]

(b) Solve the equation

$$\sin(\theta + 30°) - \cos(\theta + 60°) = 1$$

where $-180° \leqslant \theta \leqslant 180°$ [8]

6 The rate at which a body loses speed S m s^{-1} as it travels through a resistive medium at time t seconds is proportional to the square of its speed at that time.

(i) Model this by a differential equation. [2]

(ii) If its initial speed is 100 m s^{-1} and after 3 seconds its speed is 60 m s^{-1}, find how much longer it will take to reduce its speed to 30 m s^{-1} [9]

7 The function h is defined as

$$\mathrm{h}(x) = \frac{x+2}{x-3} \qquad x \in \mathbb{R} \quad x \neq 3$$

(i) Find the inverse function $\mathrm{h}^{-1}(x)$ stating its domain. [6]

(ii) Rewrite $\frac{x+2}{x-3}$ in the form $a + \frac{b}{x-3}$ [3]

(iii) Write down two functions $\mathrm{f}(x)$ and $\mathrm{g}(x)$ such that $\mathrm{h}(x) = \mathrm{fg}(x)$.
Neither $\mathrm{f}(x)$ nor $\mathrm{g}(x)$ may be the identity function. [2]

8 Part of the graph of

$$2\sin x - x = \tan y$$

is shown in **Fig. 1** below.

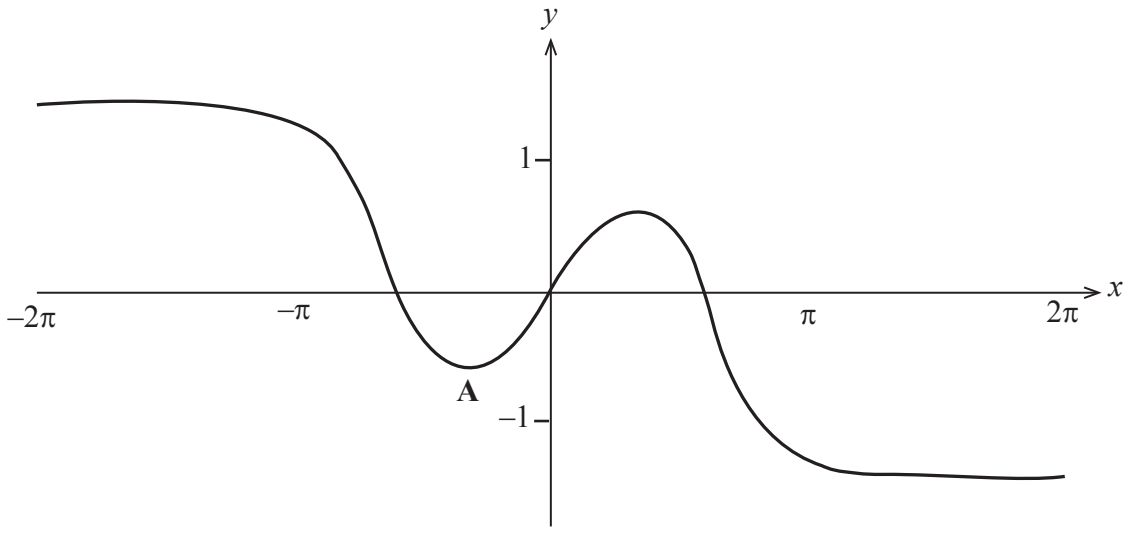

Fig. 1

(i) Find $\frac{dy}{dx}$ [4]

(ii) Hence find the coordinates of the turning point labelled **A** in **Fig. 1** above. [5]

(iii) State the equations of the 2 horizontal asymptotes. [2]

THIS IS THE END OF THE QUESTION PAPER