



ADVANCED  
General Certificate of Education  
2013

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**Mathematics**  
Assessment Unit F2  
*assessing*  
Module FP2: Further Pure Mathematics 2  
**[AMF21]**



**THURSDAY 30 MAY, MORNING**

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**TIME**

1 hour 30 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided.  
Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

**INFORMATION FOR CANDIDATES**

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all seven questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1 (i)** Show that

$$\sum_{r=1}^n (2r-1)^2 = \frac{n}{3}(4n^2-1)$$

[4]

**(ii)** Using this result, evaluate

$$\sum_{r=5}^{40} (2r-1)^2$$

[3]

**2** Find, in terms of  $\pi$ , the general solution of the equation

$$\tan^4 x - 4 \tan^2 x + 3 = 0$$

[6]

**3 (i)** Using Maclaurin's theorem, show that a series expansion for  $\ln(1+ax)$ , where  $a$  is a constant, up to and including the term in  $x^3$  is

$$\ln(1+ax) = ax - \frac{a^2 x^2}{2} + \frac{a^3 x^3}{3}$$

[6]

**(ii)** Hence or otherwise find a series expansion for

$$\ln \left\{ \frac{(1-3x)^2}{1+2x} \right\}$$

up to and including the term in  $x^3$

[5]

**(iii)** For the series expansion in **(ii)** find the coefficient of  $x^n$

[2]

4 A man adds 10 grams of sugar to a cup of hot tea. The mass of sugar,  $M$  grams, undissolved in the tea  $t$  seconds after being added is modelled by the differential equation

$$\frac{dM}{dt} + \frac{2M}{10-t} = \frac{1}{5} \quad 0 \leq t < 10$$

(i) Find  $M$  in terms of  $t$ . [9]

(ii) Find the mass of sugar undissolved after 5 seconds. [2]

5 Use the principle of mathematical induction to show that for  $n \geq 1$

$$\sum_{r=1}^n r^2 \geq \frac{n(n+1)^2}{4} \quad [8]$$

6 (i) Show that an equation of the tangent to the parabola  $y^2 = 4ax$  at the point  $P(at^2, 2at)$  is

$$x - ty + at^2 = 0 \quad [4]$$

(ii) Find the coordinates of the point  $Q$  where this tangent cuts the  $y$ -axis. [2]

(iii) If  $O$  is the origin find the point of intersection of the perpendicular bisectors of the lines  $PQ$  and  $OQ$ . [5]

(iv) As  $t$  varies find the equation of the locus of this point of intersection. [3]

7 (i) Using De Moivre's theorem, show that if  $z = \cos \theta + i \sin \theta$  then

$$2 \cos n\theta = z^n + \frac{1}{z^n} \quad [3]$$

(ii) Hence or otherwise show that if  $z = \cos \theta + i \sin \theta$  the equation

$$3z^4 - 5z^3 + 8z^2 - 5z + 3 = 0$$

can be transformed into the equation

$$6 \cos^2 \theta - 5 \cos \theta + 1 = 0 \quad [4]$$

(iii) Using (ii) or otherwise solve the equation

$$3z^4 - 5z^3 + 8z^2 - 5z + 3 = 0 \quad [9]$$

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**THIS IS THE END OF THE QUESTION PAPER**

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