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ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit F3

assessing

Module FP3: Further Pure Mathematics 3

[AMF31]

TUESDAY 10 JUNE, MORNING

AMF31

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A cube ABCDPQRS, of side 2 units, is centred on the origin as shown in **Fig. 1** below.

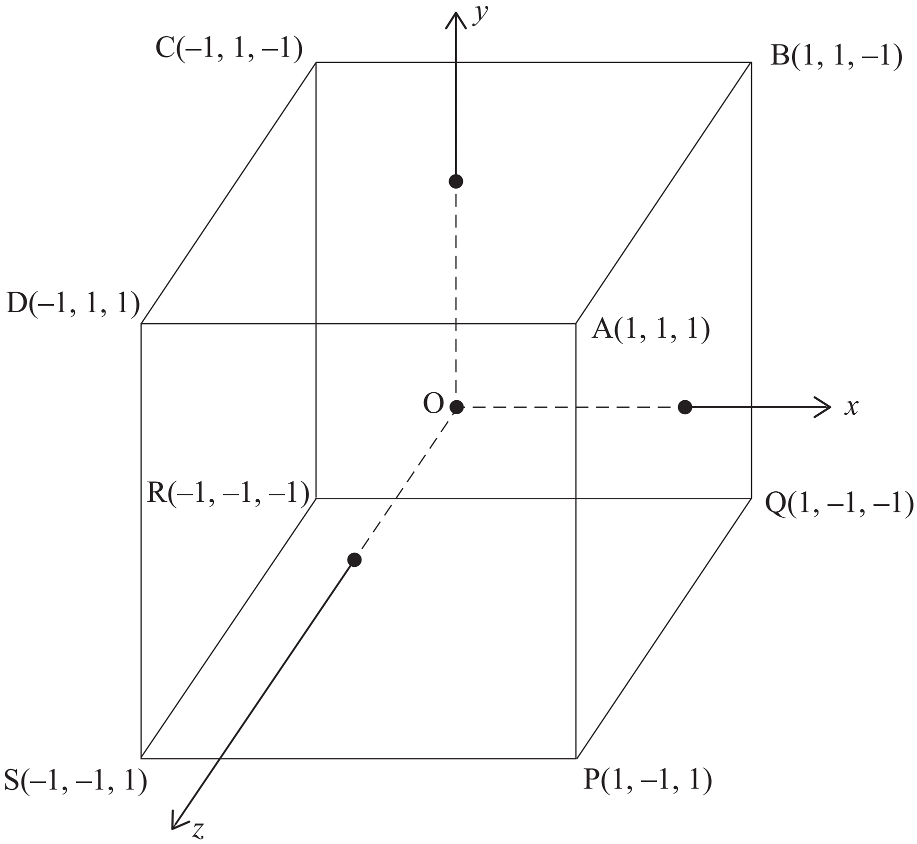

Fig. 1

Showing your method, find which of the points A, B, C, D, P, Q, R and S lie on:

(i) the line whose equation is $\left\{\mathbf{r} - \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}\right\} \times \begin{pmatrix} -1 \\ -1 \\ 0 \end{pmatrix} = \mathbf{0}$ [3]

(ii) the plane whose equation is $\mathbf{r}.\begin{pmatrix} 0 \\ 1 \\ -1 \end{pmatrix} = 0$ [3]

2 A right triangular prism includes vertices:
A(3, 4, 4)
B(1, 2, 1)
C(2, 7, 3)
D(10, 6, 5)
as shown in **Fig. 2** below:

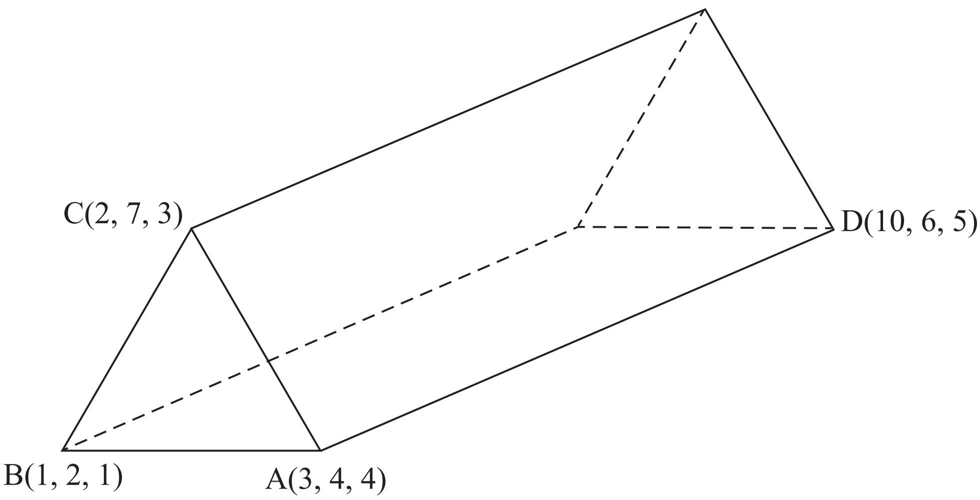

Fig. 2

Using a scalar triple product, find the volume of the prism. [6]

3 **(a)** Differentiate

$$\sin^{-1}(\sqrt{1-x^2})$$

simplifying your answer. [4]

(b) Show that

$$\frac{d}{dx}\tan^{-1}(\sinh x) = \operatorname{sech} x$$ [4]

4 Three lines are defined by:

$$\begin{aligned} &L_1 \quad \mathbf{r} = \mathbf{i} + \mathbf{j} - 3\mathbf{k} + \lambda(2\mathbf{i} - \mathbf{j} + \mathbf{k}) \\ &L_2 \quad \mathbf{r} = 2\mathbf{i} - \mathbf{j} + 7\mathbf{k} + \mu(\mathbf{i} + \mathbf{j} - 2\mathbf{k}) \\ &L_3 \quad \frac{x-2}{3} = \frac{y+1}{2} = \frac{z-7}{4} \end{aligned}$$

(i) Find whether or not the lines L_1 and L_2 are skew. [5]

(ii) Find a vector equation of a plane containing the lines L_2 and L_3 [6]

5 By using the substitution $u = x + \frac{1}{2}$, or otherwise, find the exact value of

$$\int_0^1 \frac{x}{x^2 + x + 1}\,dx$$ [8]

6 **(a)** Show that

$$\int_3^6 \frac{1}{2x+6}\sqrt{\frac{x+3}{x-3}}\,dx = \frac{1}{2}\ln\left(2 + \sqrt{3}\right)$$ [5]

(b) **(i)** Using the exponential definitions of $\sinh x$ and $\cosh x$, prove that

$$\coth^2 x - \operatorname{cosech}^2 x \equiv 1$$ [5]

(ii) Hence, or otherwise, find in logarithmic form the solution of the equation

$$3\coth^2 x - 8\operatorname{cosech} x + 1 = 0$$ [6]

7 **(i)** Show that

$$\frac{\mathrm{d}}{\mathrm{d}x}\{\sin^{n-1}x\cos x\} = (n-1)\sin^{n-2}x - n\sin^{n}x$$

where $n \geqslant 2$ [3]

Let $I_n = \int \mathrm{e}^{2x}\sin^n x\,\mathrm{d}x \quad$ for $n \geqslant 0$

(ii) Prove that

$$\left(1+\frac{n^2}{4}\right)I_n = \tfrac{1}{2}\mathrm{e}^{2x}\sin^n x - \tfrac{1}{4}n\,\mathrm{e}^{2x}\sin^{n-1}x\cos x + \tfrac{1}{4}n(n-1)I_{n-2}$$

for $n \geqslant 2$ [9]

(iii) A wine glass can be modelled by rotating the curve

$$y = \mathrm{e}^x\sin^2 x$$

through 2π radians about the x-axis between the ordinates $x = -\dfrac{\pi}{2}$ and $x = \dfrac{\pi}{2}$

Show that the maximum volume that the wine glass can contain is $\dfrac{13}{20}\pi\sinh\pi$. [8]

THIS IS THE END OF THE QUESTION PAPER