



Rewarding Learning

ADVANCED
General Certificate of Education
2014

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]



MONDAY 16 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ ms}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 **Fig. 1** below shows two light inextensible strings AB and PC used to keep a small smooth ring hanging in equilibrium. AB passes through the ring and PC is attached to the ring at P. A is attached to a fixed point on a vertical wall and B is attached to a fixed point on a horizontal ceiling. The ring can be modelled as a particle of weight 40 N.

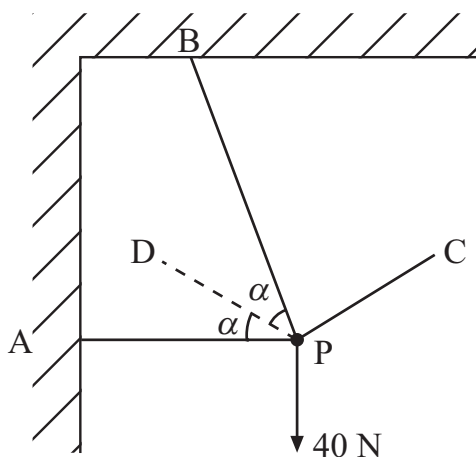


Fig. 1

AP is horizontal and the angle APB is 2α , where $\cos \alpha = 0.8$
PD bisects the angle APB.

- (i) Explain briefly why the resultant, R , of the forces in AP and PB acts along PD. [1]

The tension in AP is S newtons.

- (ii) Find, in terms of S , the magnitude of R . [2]

The tension, T , in PC is a minimum when PC makes an angle θ with the horizontal.

- (iii) Explain briefly why $\sin \theta = 0.8$ [2]

- (iv) Find the minimum value of T . [3]

- 2 The forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 newtons act on a particle, P, moving it along the line whose vector equation is

$$\mathbf{r} = \lambda \mathbf{b}$$

where λ is a scalar constant and \mathbf{b} is measured in metres.

Given that

$$\mathbf{F}_1 = \begin{pmatrix} 3 \\ -5 \\ 4 \end{pmatrix} \quad \text{and} \quad \mathbf{b} = \begin{pmatrix} 3 \\ 5 \\ 4 \end{pmatrix}$$

- (i) show that the work done by \mathbf{F}_1 as P is moved along the line is 0 J. [2]

- (ii) Explain what the result in (i) says about the directions of \mathbf{F}_1 and \mathbf{b} . [1]

A and B are the two points on the line where λ takes the values 0 and 2 respectively. The total work done by the three forces as the particle moves from A to B is W joules.

Given that

$$\mathbf{F}_2 = \begin{pmatrix} 8 \\ 2a \\ 9 \end{pmatrix} \quad \text{and} \quad \mathbf{F}_3 = \begin{pmatrix} -a \\ a \\ -a \end{pmatrix}$$

where a is a scalar constant,

- (iii) find W in terms of a . [4]

- (iv) If $W = 200$, find a . [1]

- 3 An engineering component is made from a rectangular metal lamina ABCD of width 32 cm and height 60 cm.

Fig. 2 below shows the first stage of production where an isosceles triangle EHF of base 16 cm and sides 17 cm is cut from the lamina with $BE = FC = 8$ cm.

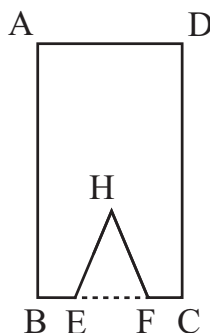


Fig. 2

- (i) If the mass of metal that is removed is 0.2 kg show that the mass remaining is 3 kg. [3]
- (ii) Show that the centre of mass at the end of this first stage is $1\frac{2}{3}$ cm above the centre of the original lamina ABCD. [5]
- (iii) If the lamina is freely suspended from A, find the angle that AB makes with the vertical. [3]

At the second stage of production the corners at A and D are removed by cutting off two identical right angled triangles with perpendicular sides 8 cm and 15 cm.

The right angles in the triangles coincide with the right angles at A and D.

The centre of mass is now at the centre of the original lamina ABCD.

Fig. 3 below shows the correct shape of the component after the second stage of production together with an incorrect shape.

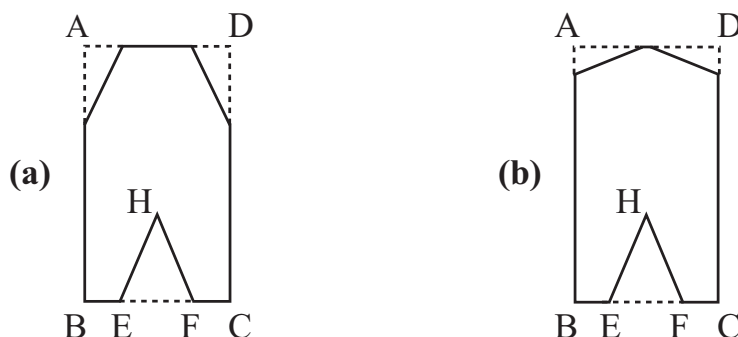


Fig. 3

- (iv) State which shape is the correct one, giving a brief explanation for your choice. [2]

- 4 The X-potential motor has been developed by aliens for a new spacecraft. The thrust $T(x)$ meganewtons (i.e. 10^6 newtons) produced by the motor is given by

$$T(x) = e^{x-1} + e^{1-x}$$

where x metres is the distance travelled from rest.

The motor is used to propel a two tonne spacecraft from a position of rest in gravity free space but needs to act only for the first two metres of travel.

- (i) Show that the work done by the motor over the first 2 m travelled by the spacecraft is approximately 4.70×10^6 J. [5]
- (ii) Find the speed of the spacecraft when it has travelled the first two metres. [2]

- 5 A particle P of mass 2 kg is attached to one end of a light elastic string of natural length 5 m and modulus of elasticity 16 g N. The other end of the string is attached to a fixed point A on a rough horizontal surface. P is held at a point B on the surface with the string just taut. It is then given a speed of 21 m s^{-1} in the direction AB.

- (i) Evaluate, in terms of d , the work done on the particle by the tension in the string as P moves a distance d metres beyond B. [2]

The coefficient of friction between the particle and the surface is 0.5

- (ii) Use the Work–Energy Principle to find how far P moves beyond B, before it comes to rest. [6]
- (iii) Determine whether this position of rest is instantaneous or permanent. [2]

- 6 Take Glasgow airport to be 200 km NE of Belfast airport.
Planes on the Belfast–Glasgow route operate at 400 km h^{-1} in still air.
One day the wind was blowing from $\text{W}15^\circ \text{N}$ at 50 km h^{-1}
The pilot set the course on the Belfast–Glasgow route at θ° anti-clockwise from its intended flight path as shown in **Fig. 4** below.



Fig. 4

- (i) Explain briefly why the plane's course was set in this direction. [1]

The plane covered the flight path at $v \text{ km h}^{-1}$

- (ii) By considering the velocity diagram for the flight show that

$$v^2 - 50v - 157500 = 0 \quad \text{equation (a)} \quad [5]$$

- (iii) Find the time taken to fly from Belfast to Glasgow. [2]

The plane was unable to land at Glasgow and immediately had to return to Belfast.
The return flight path was covered at $w \text{ km h}^{-1}$

- (iv) By considering the velocity diagram for this flight show that

$$w^2 + 50w - 157500 = 0 \quad \text{equation (b)} \quad [3]$$

- (v) Show that if $w = -v$ then equation (b) transforms to equation (a). [1]

The plane left Belfast at noon.

- (vi) Find the time at which it landed back in Belfast. [2]

- 7 **Fig. 5** below shows a particle of mass m attached to one end of each of two elastic strings, S_1 and S_2 . The other ends of the strings are attached to fixed points on a smooth horizontal surface – S_1 to A and S_2 to B.

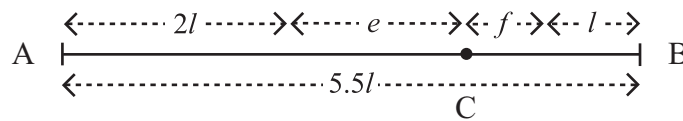


Fig. 5

S_1 has natural length $2l$ and modulus of elasticity λ .

S_2 has natural length l and modulus of elasticity 2λ .

The distance AB is $5.5l$

When the particle is in equilibrium at C the extension in S_1 is e and the extension in S_2 is f .

- (i) Show that $e = 2l$ [5]

The particle is displaced a small distance d to the right of C so that both strings remain taut. It is released from rest where $d < 0.25l$

At time t its displacement from C is x .

- (ii) Show that the equation of motion, in terms of x , is of the standard form for S.H.M. [5]

- (iii) Given that $\lambda = 6.4ml\pi^2$, show that the period of the motion is 0.5 [2]

The particle is released from rest at $t = 0$ and $x = 0.2l$

- (iv) Write down in its simplest form the expression for x as a function of time. [1]

- (v) Briefly explaining your answer, state the maximum distance the particle could be displaced to the right of C and still move in S.H.M. [2]

THIS IS THE END OF THE QUESTION PAPER

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