Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2014

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

AMF11

TUESDAY 24 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{A} = \begin{pmatrix} 2 & -5 \\ -1 & 3 \end{pmatrix}$ and $\mathbf{C} = \begin{pmatrix} 1 & 0 \\ 4 & 2 \end{pmatrix}$

Given that $\mathbf{ABC} = \mathbf{I}$, find the matrix $\mathbf{B}$ [5]

2 The equations of two circles C_1 and C_2 are

$$C_1 \qquad x^2 + y^2 + 2x - 6y - 15 = 0$$

$$C_2 \qquad x^2 + y^2 + 2y - 3 = 0$$

(i) Find the points of intersection of C_1 and C_2 [8]

A third circle C_3 is defined as

$$C_3 \qquad 4x^2 + 4y^2 - 4x - 16y - 127 = 0$$

(ii) Show that circle C_2 lies entirely inside circle C_3 [8]

3 A binary operation $*$ is defined on the set of all ordered pairs (x, y) of real numbers.
The operation is given as

$$(a, b) * (c, d) = (ac, b + d + 2)$$

(i) Show that $*$ is associative. [4]

(ii) Find the identity element. Justify your answer. [4]

4 **(i)** Show that the determinant of

$$\begin{pmatrix} 5 & -2 & -a \\ 4 & 2 & -6 \\ 1 & a & -4 \end{pmatrix}$$

is $-4a^2 + 32a - 60$ [3]

Consider the system of linear equations

$$\begin{aligned} 5x - 2y - az &= 3 \\ 4x + 2y - 6z &= 2 \\ x + ay - 4z &= 0 \end{aligned}$$

where x, y and z are real numbers.

(ii) Determine the number of solutions for the above system of equations for **each** of the cases:

$$a = 3$$
$$\text{and } a = 4$$ [7]

(iii) Find the general solution of this system of equations when $a = 5$ [4]

5 **(a)** The transformation represented by the 2×2 matrix **M** maps the points (2, 3) and (5, −1) onto (7, 2) and (9, 5) respectively.

Find the matrix **M** [5]

(b) The set of points which form the curve

$$x^2 + 5y^2 + 4xy - 6x - 8y = 0$$

is mapped by the matrix $\mathbf{N} = \begin{pmatrix} 0 & -1 \\ 1 & 2 \end{pmatrix}$

Show that the curve formed by the image points is a circle with radius $\sqrt{13}$ [9]

6 **(a)** Let $z_1 = \sqrt{3} + \mathrm{i}$ and $z_2 = 1 - \mathrm{i}$

(i) Find the modulus and argument of each of the complex numbers z_1 and z_2 [6]

(ii) Verify that $|z_1 z_2| = |z_1||z_2|$ [4]

(b) **(i)** Sketch on an Argand diagram the locus of those points u which satisfy

$$|u - (4 + 15\mathrm{i})| = |u + \mathrm{i}|$$ [3]

(ii) On the same diagram sketch the locus of those points v which satisfy

$$\arg\{v - (2 + 7\mathrm{i})\} = \frac{\pi}{6}$$ [3]

(iii) On your diagram shade the region which represents the locus of those points w, where w satisfies both

$$|w - (4 + 15\mathrm{i})| \leqslant |w + \mathrm{i}|$$

$$\text{and } \frac{\pi}{6} \leqslant \arg\{w - (2 + 7\mathrm{i})\} \leqslant \pi$$ [2]

THIS IS THE END OF THE QUESTION PAPER