



ADVANCED
General Certificate of Education
2014

Mathematics
Assessment Unit C4
assessing
Module C4: Core Mathematics 4
[AMC41]



THURSDAY 22 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 (i) Write

$$\frac{2x - 7}{(5 - x)(1 + x)}$$

in partial fractions.

[6]

(ii) Hence find

$$\int \frac{2x - 7}{(5 - x)(1 + x)} \, dx \quad [4]$$

2 The points A and B have position vectors:

$$\begin{aligned} \vec{OA} &= 3\mathbf{i} - 4\mathbf{j} \\ \text{and} \quad \vec{OB} &= 7\mathbf{i} + 5\mathbf{j} \end{aligned}$$

relative to a fixed origin O.

(i) Find \vec{AB} .

[2]

The point C has position vector

$$\vec{OC} = 3\mathbf{i} - 2\mathbf{j}$$

(ii) Find a vector equation of the line through C, parallel to AB.

[3]

(iii) Show that the point with position vector $(11\mathbf{i} + 16\mathbf{j})$ lies on this line.

[4]

3 Use the substitution $u = 3x - 5$ to evaluate

$$\int_2^3 6x\sqrt{3x-5} \, dx \quad [9]$$

4 The curved surface of a glass bowl can be modelled by rotating the curve

$$y = e^x + 1$$

between the lines $x = 0$ and $x = 1$ through 2π radians about the x -axis.

(i) Find the maximum volume that the bowl can hold. [7]

(ii) State one assumption made in the modelling. [1]

5 At time $t = 0$ hours a small block of ice starts to melt.

The volume, $V \text{ cm}^3$, of the solid ice decreases with time, at a rate which is proportional to the square root of the volume of ice remaining at that time.

This can be modelled by the differential equation

$$\frac{dV}{dt} = k\sqrt{V}$$

Initially the volume of ice is 64 cm^3 and one hour later the volume of ice is 48 cm^3

If the ice started to melt at 12.00 (noon), find the time at which the ice has completely melted. [10]

6 The expression $(7 \sin x - 24 \cos x)$ can be written in the form

$$R \sin(x - \alpha)$$

where R is an integer and $0 \leq \alpha \leq \frac{\pi}{2}$

(i) Find R and α . [4]

(ii) Hence find

$$\int \frac{1}{(7 \sin x - 24 \cos x)^2} dx \quad [3]$$

7 The function f is defined as

$$f: x \rightarrow \tan x \quad -\frac{\pi}{4} \leq x \leq \frac{\pi}{4}$$

(i) Write down the inverse function f^{-1} and state its domain and range. [4]

The function g is defined as

$$g: x \rightarrow |x| \quad x \in \mathbb{R}$$

(ii) Find the composite function gf , stating its range. [4]

(iii) Hence sketch the graph of $y = gf(x)$ [3]

8 The parametric equations of a curve are

$$x = 2t - \sin 2t \quad y = 4 \cos t$$

where $\frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$

Find the **exact** coordinates of the point on the curve at which the gradient is $\sqrt{2}$ [11]

THIS IS THE END OF THE QUESTION PAPER
