Rewarding Learning

ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit C3

assessing

Module C3: Core Mathematics 3

AMC31

[AMC31]

WEDNESDAY 13 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Use Simpson's Rule with four strips to find an approximate value for

$$\int_{0.1}^{0.5} \sin x \cos x \, dx$$ [6]

2 A number of goldfish were introduced into a garden pond.
After t years the number of goldfish, N, can be modelled by the equation

$$N = 5e^{0.25t} \qquad t \geqslant 0$$

(i) Find the number of goldfish that were introduced into the garden pond. [1]

(ii) Find the number of complete years it would take for the number of goldfish to double. [3]

(iii) Evaluate $\frac{dN}{dt}$ when $t = 4$ and state what this value represents. [4]

3 **(a)** Simplify

$$\frac{12x + 18}{2x^2 + 5x + 3} - \frac{2x + 3}{x + 1}$$ [4]

(b) Write

$$\frac{x^2 - 3}{(x + 2)(x + 1)^2}$$

in partial fractions. [6]

4 In the binomial expansion, in ascending powers of x, of

$$(1 + ax)^n$$

the coefficients of x and x^2 are $-\frac{1}{4}$ and $-\frac{1}{32}$ respectively.

Find the values of a and n. [7]

5 A child's drawing of a fish is shown in **Fig. 1** below.

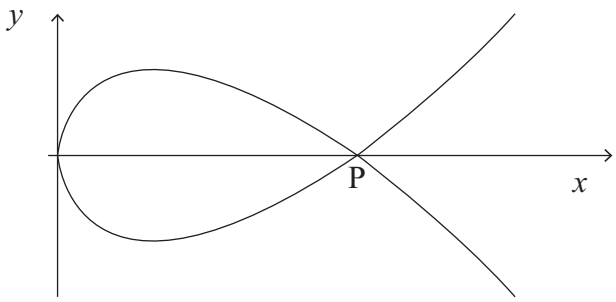

Fig. 1

P is a point where the curve crosses the x-axis.
The curve can be modelled by the parametric equations

$$x = 3t^2 \qquad y = 3t - t^3$$

The corresponding Cartesian equation of the curve has the form

$$y^2 = ax^3 + bx^2 + cx$$

(i) Find a, b and c. [4]

(ii) Find P. [3]

6 Find the equation of the normal to the curve

$$y = x \ln x^2$$

at the point where $x = -1$ [8]

7 **(a)** **(i)** Sketch the graph of

$$y = \left|\tan\frac{\theta}{2}\right|$$

where $0 \leqslant \theta \leqslant 4\pi$ [2]

(ii) Find the exact solutions of

$$\left|\tan\frac{\theta}{2}\right| = 1$$

where $0 \leqslant \theta \leqslant 4\pi$ [4]

(b) **(i)** Prove the identity

$$\frac{\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1} \equiv \sec\theta\tan\theta$$ [6]

(ii) Hence solve the equation

$$\frac{\operatorname{cosec}\theta}{\operatorname{cosec}^2\theta - 1} = 2\tan\theta$$

where $-\pi < \theta < 0$ [6]

8 **(a)** Find

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{\cot^2 x}{3x-2}\right)$$ [6]

(b) Find $$\int \frac{1}{\cos^2 4x} + \frac{3}{(3-4x)} + 5\mathrm{e}^x + \frac{\tan x}{\cos x}\,\mathrm{d}x$$ [5]

THIS IS THE END OF THE QUESTION PAPER