



Rewarding Learning
ADVANCED
General Certificate of Education
2015

Mathematics

Assessment Unit C4
assessing
Module C4: Core Mathematics 4



[AMC41]
TUESDAY 26 MAY, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the angle between the two lines whose vector equations are

$$\mathbf{r}_1 = \mathbf{i} + \mathbf{j} - \mathbf{k} + \mu(2\mathbf{i} - 3\mathbf{j})$$

$$\mathbf{r}_2 = \mathbf{i} - \mathbf{j} + 3\mathbf{k} + \lambda(\mathbf{i} - \mathbf{j} - \mathbf{k}) \quad [5]$$

2 Using the substitution $u = x - 1$ or otherwise find

$$\int_2^5 \frac{x-2}{\sqrt{x-1}} dx \quad [6]$$

3 (a) A curve is given by the parametric equations

$$x = e^{2t} \quad y = 1 + e^t$$

Find the gradient of the curve when $t = 0$ [5]

(b) Find the equation of the normal to the curve

$$2x^2 + y^2 - 3y = 0$$

at the point $(1, 2)$. [9]

4 The number of yeast cells increases at a rate proportional to the number of yeast cells, N , present at any time t .

This can be modelled by the differential equation

$$\frac{dN}{dt} = 0.02N$$

If the number of yeast cells present at time $t = 0$ minutes is N_0 , find the length of time it will take for the number of yeast cells to be $2N_0$ [8]

5 (a) Sketch the graph of

$$y = \tan^{-1} x$$

where $-\frac{\pi}{2} < y < \frac{\pi}{2}$ [2]

(b) Solve the equation

$$\tan 2\theta = 3 \cot \theta$$

where $0^\circ \leq \theta \leq 360^\circ$ [8]

6 The maximum volume of liquid that a chinese wok can hold can be modelled by the volume generated by the rotation of the curve

$$x^2 + y^2 = a^2$$

through 2π radians about the x -axis between $x = \frac{a}{2}$ and $x = a$ as shown in **Fig. 1** below.

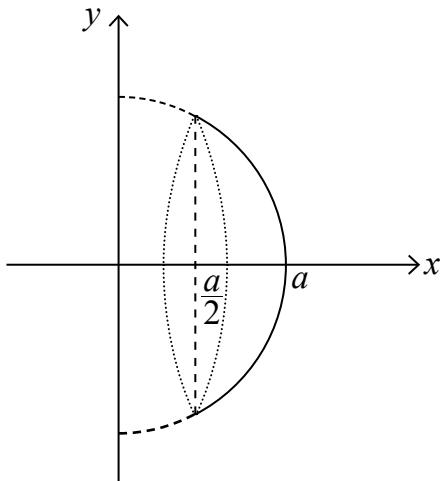


Fig. 1

(i) Find the volume generated by the rotation. [7]

(ii) Given that when the wok is 'full' it holds $2880\pi \text{cm}^3$, find a . [2]

7 A function f is defined by

$$f : x \rightarrow \frac{1}{x+2} \quad x \in \mathbb{R} \quad x > -2$$

(i) Sketch the graph of $y = f(x)$. [1]

(ii) State the equations of the asymptotes to this graph. [2]

A function g is defined by

$$g : x \rightarrow \frac{1}{x-1} \quad x \in \mathbb{R} \quad x > 3$$

(iii) Find the range of g . [1]

(iv) Find the composite function fg . [2]

(v) Find the inverse function $(fg)^{-1}$ [4]

8 (a) Find

$$\int x^2 \ln x^2 \, dx \quad [6]$$

(b) Find the exact value of

$$\int_0^{\frac{\pi}{4}} \cos^2 x \sin^3 x \, dx \quad [7]$$

THIS IS THE END OF THE QUESTION PAPER
