



Rewarding Learning

ADVANCED

General Certificate of Education

2015

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

WEDNESDAY 24 JUNE, MORNING



TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all seven** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or a scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1** Three spheres A, B and C of equal radius and masses m , $4m$ and $2m$ respectively are at rest in a straight line on a smooth horizontal surface. A is projected towards B with speed u and collides directly with it. As a result of the collision A is brought to rest.

(i) Show that the coefficient of restitution between A and B is $\frac{1}{4}$ [5]

A collision now occurs between B and C.

The coefficient of restitution between B and C is $\frac{4}{5}$

(ii) Find, in terms of u , the velocities of B and C after this collision. [4]

(iii) Find, in terms of u and m , the total loss of kinetic energy due to the two collisions. [3]

- 2 **Fig. 1** below shows a framework of five light pin-jointed rods AB, BC, CD, DA and DB. The framework supports a weight of 20 N at A. The rods all lie in the same vertical plane. $BC = CD = DA = DB = 50$ cm. The rod AD is horizontal. The framework is freely hinged to a vertical wall at B and the pin at C rests against the wall, making a smooth contact.

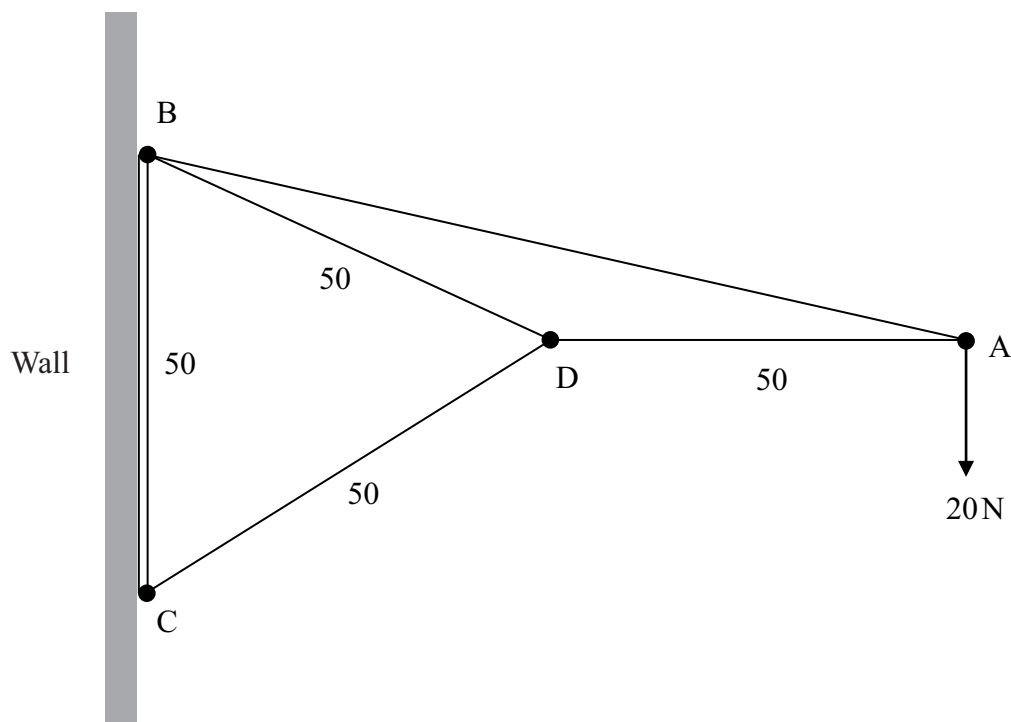


Fig. 1

- (i) Explain briefly why the reaction on the pin at C must be horizontal. [1]
- (ii) Show that the magnitude of the reaction at C is approximately 37.3 N. [3]
- (iii) Find the forces in the rods CD, CB, DA and AB. [5]
- (iv) State which of these rods could be replaced by light ropes, giving a reason for your answer. [2]

- 3 **Fig. 2** below shows the relative positions of the Moon, M, Earth, E, and Sun, S, represented as particles, lying in a straight line with the Earth between the Sun and the Moon during a lunar eclipse.
 The distance between S and E is R .
 The distance between E and M is r .

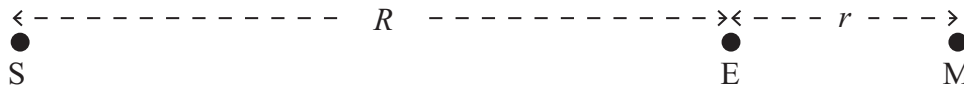


Fig. 2

The masses of the Sun, Earth and Moon are M_S , M_E and M_M respectively.

- (i) Write down the expression, in terms of M_S , M_M , R , r and G , the universal gravitational constant, for the gravitational force on the Moon due to the Sun. [1]

The magnitude of the gravitational force between E and M is F_{EM}

The magnitude of the gravitational force between S and M is F_{SM}

- (ii) Show that $\frac{F_{EM}}{F_{SM}} = \frac{M_E}{M_S} \left(\frac{R}{r} + 1 \right)^2$ [2]

Given that $M_S = 3.328 \times 10^5 M_E$ and $R = 3.896 \times 10^2 r$,

- (iii) find the value of $\frac{F_{EM}}{F_{SM}}$ [2]

During a solar eclipse M, E and S lie in a straight line with M between S and E.

- (iv) Find the value of $\frac{F_{EM}}{F_{SM}}$ in this case. [3]

- 4 Take \mathbf{i} and \mathbf{j} to be unit vectors in the directions of the axes Ox and Oy respectively.

Fig. 3 below shows three forces \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 newtons which act at the points whose coordinates are $(2, 5)$, $(3, -4)$ and $(-2, 2)$ respectively.

$$\mathbf{F}_1 = (-2a \mathbf{i} + 3\mathbf{j})$$

$$\mathbf{F}_2 = (-2\mathbf{i} - 4a\mathbf{j})$$

$$\mathbf{F}_3 = -6\mathbf{i}$$

where a is a constant and the unit of distance is metres.

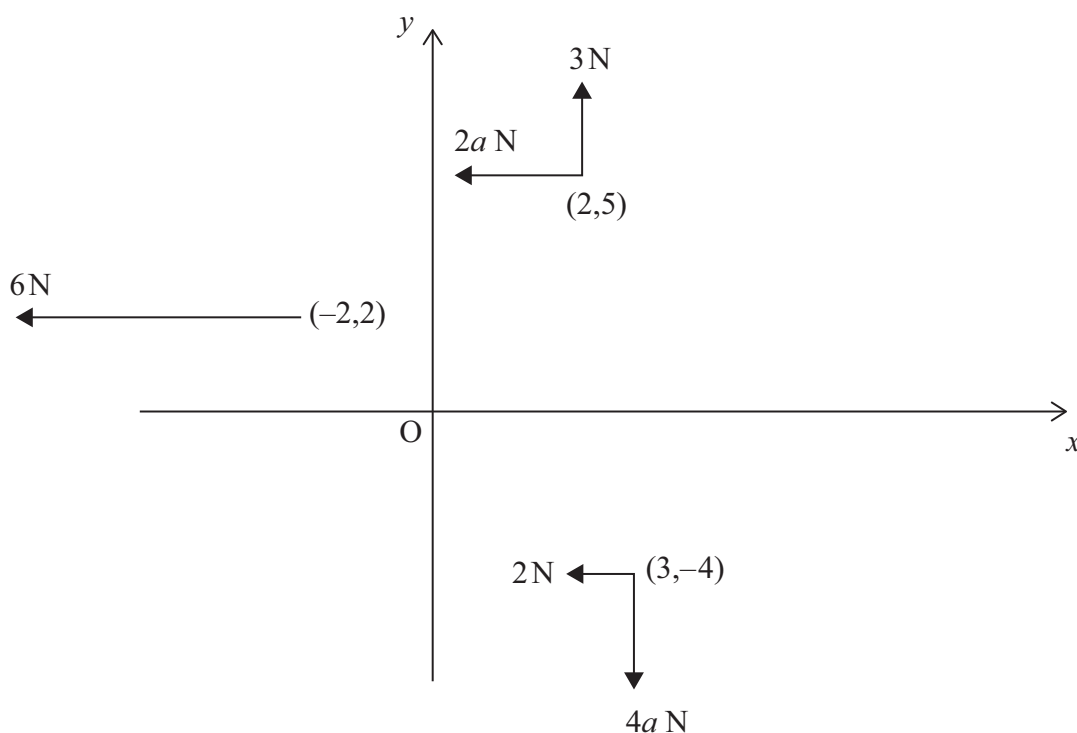


Fig. 3

- (i) Show that the sum of the moments of \mathbf{F}_1 , \mathbf{F}_2 and \mathbf{F}_3 about the origin can be expressed as $(10 - 2a)$ N m anticlockwise. [4]
- (ii) If the system of forces reduces to a single force acting at the origin, write down the value of a . [1]
- (iii) If, instead, the system of forces is equivalent to a force acting at the origin together with a couple of magnitude 16 N m, find the possible values of a . [3]

- 5 (i) Show that the centre of mass, G , of a uniform solid right circular cone of height h and base radius r lies on its axis of symmetry $\frac{h}{4}$ from its base. [6]

Fig. 4 below shows a uniform solid right circular cone of height 15 cm and base radius 8 cm resting on a horizontal surface which is sufficiently rough to prevent slipping. A force P newtons, which just causes the cone to topple, is applied to the vertex of the cone at 20° below the horizontal.

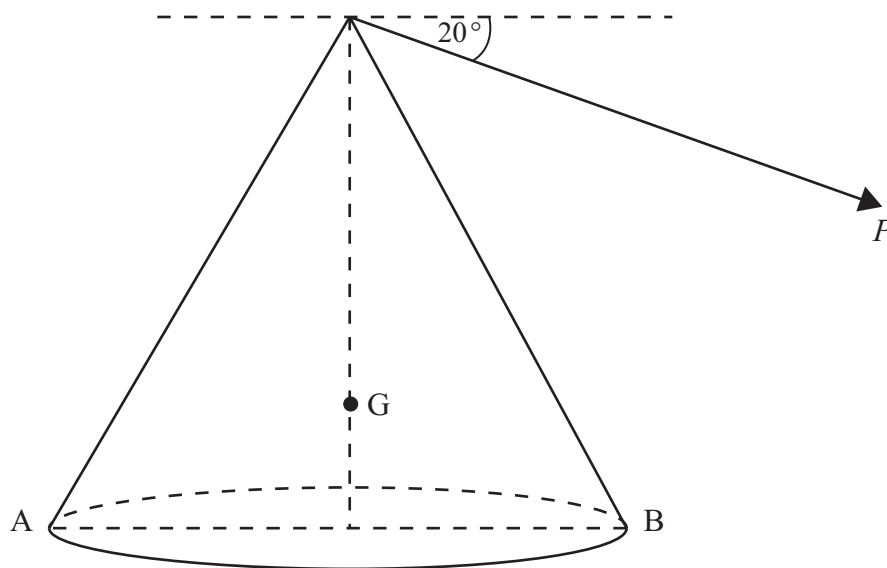


Fig. 4

A diameter of the base, AB , the centre of mass, G , and the force P all lie in the same vertical plane.

The mass of the cone is $\frac{1}{2}$ kg.

- (ii) Find P . [4]

- 6 The velocity v of surface water waves depends on the wavelength λ of the waves, the acceleration due to gravity g , the density of water ρ and the surface tension S of water. The velocity is related to the other variables as follows:

$$v = k \lambda^a g^b \rho^c S^d$$

where k is a dimensionless constant.

The dimensions of S are $[MT^{-2}]$

Using the method of dimensions,

- (i) show that $c + d = 0$, [5]

- (ii) find a and b in terms of d , [3]

- (iii) show that a possible relationship between the variables is

$$v = k \sqrt{\lambda g} \left(\frac{S}{\rho \lambda^2 g} \right)^d$$
 [2]

For deep water waves, the velocity is independent of S .

- (iv) Using the relationship given in (iii), write down a formula for the velocity of deep water waves. [1]

For shallow water waves, the velocity is independent of g .

- (v) Again using the relationship given in (iii), find a formula for the velocity of shallow water waves. [2]

- 7 **Fig. 5** below shows a bead of mass m threaded on a smooth circular wire of radius r fixed in a vertical plane. O is the centre of the circle.
A light inextensible string attached to the bead passes through a smooth ring fixed at O and supports a particle of mass M hanging freely.
The bead is projected with speed u from A, the lowest point on the wire.
When the bead is at P, OP makes an angle θ with the downward vertical and the speed of the bead is v .

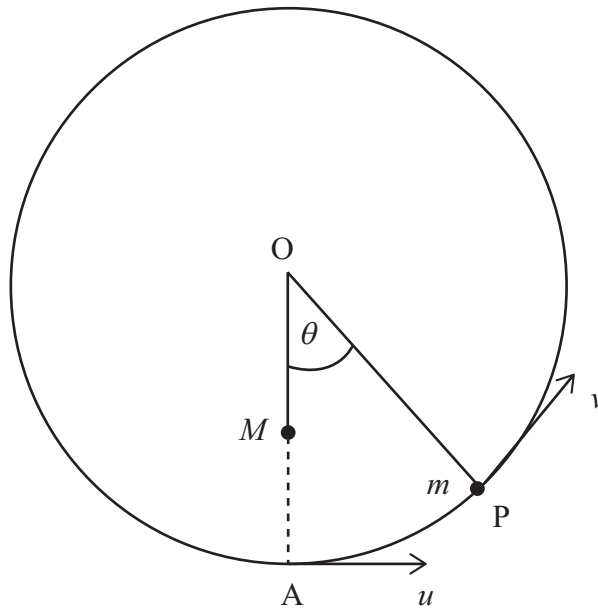


Fig. 5

Take the gravitational potential energy to be zero at A.

Given that $u^2 = 8gr$,

- (i) show that the bead makes complete revolutions, [4]
- (ii) find, in terms of M , m , g and θ , an expression for the reaction of the wire on the bead at P, [5]
- (iii) show that if there is no reaction between the bead and the wire at some points of the motion then $3m \leq M \leq 9m$. [4]

THIS IS THE END OF THE QUESTION PAPER
