Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2016

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

[AMF11]

MONDAY 27 JUNE, MORNING

MARK SCHEME

GCE Advanced/Advanced Subsidiary (AS) Mathematics

Mark Schemes

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for correct working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidates's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

		AVAILABLE MARKS
1 **(i)** $\mathbf{A}^2 = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}\begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$	M1	
$= \begin{pmatrix} 13 & 12 \\ -9 & -8 \end{pmatrix}$	W1	
$3\mathbf{A} - 2\mathbf{I} = \begin{pmatrix} 15 & 12 \\ -9 & -6 \end{pmatrix} - \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix}$	M1	
$= \begin{pmatrix} 13 & 12 \\ -9 & -8 \end{pmatrix}$	W1	
(ii) $\mathbf{A}^2 = 3\mathbf{A} - 2\mathbf{I}$		
Multiply through by $\mathbf{A}^{-1}$	M1	
$\Rightarrow \mathbf{A}^{-1}\mathbf{A}^2 = 3\mathbf{A}^{-1}\mathbf{A} - 2\mathbf{A}^{-1}\mathbf{I}$		
$\Rightarrow \mathbf{A} = 3\mathbf{I} - 2\mathbf{A}^{-1}$	W1	
$\Rightarrow 2\mathbf{A}^{-1} = -\mathbf{A} + 3\mathbf{I}$		
$\Rightarrow \mathbf{A}^{-1} = -\frac{1}{2}\mathbf{A} + \frac{3}{2}\mathbf{I}$	W1	7
2 $\begin{vmatrix} 2 & a-1 & -1 \\ a+2 & 3 & 0 \\ 2 & 3 & a+1 \end{vmatrix} = 0$	MW1	
$\Rightarrow 2(3a+3) - (a-1)[(a+2)(a+1)] - 1[3(a+2) - 6] = 0$	M1	
$\Rightarrow 6a + 6 - (a^2 - 1)(a+2) - (3a + 6 - 6) = 0$		
$\Rightarrow 3a + 6 - (a^2 - 1)(a+2) = 0$		
$\Rightarrow 3(a+2) - (a^2 - 1)(a+2) = 0$	W1	
$\Rightarrow (a+2)(3 - a^2 + 1) = 0$		
$\Rightarrow (a+2)(4 - a^2) = 0$		
$\Rightarrow (a+2)(2-a)(2+a) = 0$	MW1	
$\Rightarrow a = -2, 2$	W1	5

		AVAILABLE MARKS
3 (a) (i) $\mathbf{S} = \mathbf{NM}$	M1 M1	
$= \begin{pmatrix} 0 & 4 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$		
$= \begin{pmatrix} 8 & -4 \\ 0 & -7 \end{pmatrix}$	W1	
(ii) Area of Q = $\lvert\det \mathbf{S}\rvert \times$ Area of R	M1	
Det $\mathbf{S} = -56 - 0 = -56$	MW1	
$\Rightarrow$ Area $= 56 \times 3$		
$= 168\,\text{cm}^2$	W1	
(b) $\begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} x \\ mx \end{pmatrix} = \begin{pmatrix} x \\ mx \end{pmatrix}$	M1 M1	
Expand to give $3x - mx = x$	MW1	
and $mx = mx$		
$\Rightarrow x(2 - m) = 0$	MW1	
$\Rightarrow m = 2$		
Therefore the line is $y = 2x$	W1	11

AVAILABLE MARKS

4 **(i)** $\begin{pmatrix} 11 & 2 & 8 \\ 2 & 2 & -10 \\ 8 & -10 & 5 \end{pmatrix}\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix} = \begin{pmatrix} -9 \\ 18 \\ 18 \end{pmatrix}$ M1 W1

$= -9\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$

Hence the eigenvalue is –9 W1

$\begin{pmatrix} 11 & 2 & 8 \\ 2 & 2 & -10 \\ 8 & -10 & 5 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix} = \begin{pmatrix} 36 \\ -18 \\ 36 \end{pmatrix}$ MW1

$= 18\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$

Hence the eigenvalue is 18 W1

(ii) $\begin{pmatrix} 11 & 2 & 8 \\ 2 & 2 & -10 \\ 8 & -10 & 5 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = 9\begin{pmatrix} x \\ y \\ z \end{pmatrix}$ M1

$\Rightarrow 11x + 2y + 8z = 9x \qquad \Rightarrow 2x + 2y + 8z = 0$ ① MW1

$2x + 2y - 10z = 9y \qquad \Rightarrow 2x - 7y - 10z = 0$ ②

$8x - 10y + 5z = 9z \qquad \Rightarrow 8x - 10y - 4z = 0$ ③

① − ② $\Rightarrow 9y + 18z = 0$

$\Rightarrow y = -2z$ MW1

Using ③ $\Rightarrow 8x + 20z - 4z = 0$

$\Rightarrow 8x + 16z = 0$

$\Rightarrow x = -2z$ W1

Hence the eigenvector is $\begin{pmatrix} -2z \\ -2z \\ z \end{pmatrix} \Rightarrow \begin{pmatrix} -2 \\ -2 \\ 1 \end{pmatrix}$ W1

Therefore the unit eigenvector is $\begin{pmatrix} \frac{-2}{3} \\ \frac{-2}{3} \\ \frac{1}{3} \end{pmatrix}$ MW1

(iii) The matrix $\mathbf{U} = \begin{pmatrix} \frac{1}{3} & \frac{2}{3} & -\frac{2}{3} \\ -\frac{2}{3} & -\frac{1}{3} & -\frac{2}{3} \\ -\frac{2}{3} & \frac{2}{3} & \frac{1}{3} \end{pmatrix}$ M1 MW1

and the corresponding matrix $D = \begin{pmatrix} -9 & 0 & 0 \\ 0 & 18 & 0 \\ 0 & 0 & 9 \end{pmatrix}$ MW1 14

			AVAILABLE MARKS
5 **(i)**	$4x + 3y = 36$		
	$\Rightarrow x = 9 - \frac{3}{4}y$	MW1	
	Substitute into the equation of C_1	M1	
	$\Rightarrow \left(9 - \frac{3}{4}y\right)^2 + y^2 - 20\left(9 - \frac{3}{4}y\right) - 14y + 99 = 0$	W1	
	$\Rightarrow 81 - \frac{27}{2}y + \frac{9}{16}y^2 + y^2 - 180 + 15y - 14y + 99 = 0$		
	$\Rightarrow \frac{25}{16}y^2 - \frac{25}{2}y = 0$	MW1	
	$\Rightarrow \frac{25}{16}y(y - 8) = 0$		
	$\Rightarrow y = 0, 8$		
	$\Rightarrow x = 9, 3$	W2	
	Therefore the coordinates are P(9, 0) and Q(3, 8)		
(ii)	P(9, 0) and Q(3, 8)		
	$\Rightarrow$ Centre is the midpoint (6, 4)	MW1	
	and diameter $= \sqrt{6^2 + 8^2} = 10$, giving a radius of 5	MW1	
	Therefore the equation of the circle C_2 is given by		
	$(x - 6)^2 + (y - 4)^2 = 25$	M1 W1	
	$\Rightarrow x^2 + y^2 - 12x - 8y + 27 = 0$		
(iii)	Gradient of PQ $= -\frac{4}{3}$	M1	
	$\Rightarrow$ gradient of tangent $= \frac{3}{4}$	MW1	
	Hence equation of tangent is		
	$y - 8 = \frac{3}{4}(x - 3)$	M1	
	$\Rightarrow 4y = 3x + 23$	W1	14

AVAILABLE MARKS

6 **(i)** The identity element is c MW1

(ii) The element of a represents a reflection since it is self-inverse MW1

(iii) A subgroup of order 3 is $\{c, b, e\}$ MW2

(iv)

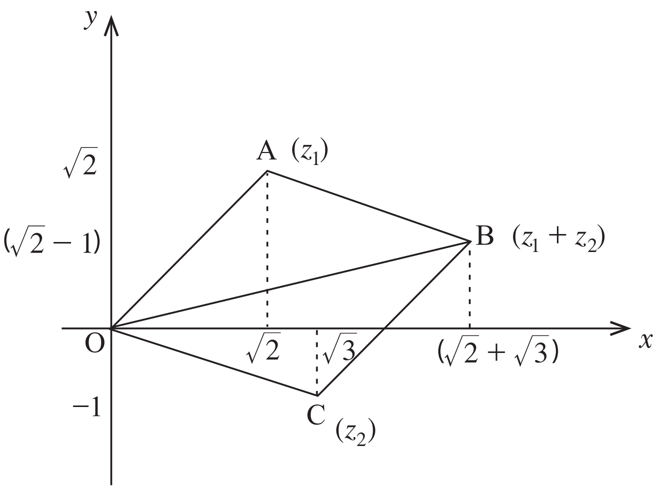
	I	p	q	r	s	t
I	I	p	q	r	s	t
p	p	I	t	s	r	q
q	q	s	I	t	p	r
r	r	t	s	I	q	p
s	s	q	r	p	t	I
t	t	r	p	q	I	s

MW2

(v) The period of the element s is 3 MW1

(vi) Any one of p, q or r is self-inverse MW1

(vii) In G the identity is c, the elements a, d, f have order 2 and b, e have order 3
In H the identity is I, the elements p, q, r have order 2 and s, t have order 3
Therefore, G and H are isomorphic with a possible isomorphism being
$c \leftrightarrow I$ MW1
$a \leftrightarrow p$
$b \leftrightarrow s$
$d \leftrightarrow r$
$e \leftrightarrow t$
$f \leftrightarrow q$ MW1 10

			AVAILABLE MARKS
7	**(i)** $\lvert z_1 \rvert = \sqrt{2+2} = 2$	M1 W1	
	$\lvert z_2 \rvert = \sqrt{3+1} = 2$	MW1	
	$\arg z_1 = \tan^{-1}\dfrac{\sqrt{2}}{\sqrt{2}} = \dfrac{\pi}{4}$	M1 W1	
	$\arg z_2 = \tan^{-1}\dfrac{-1}{\sqrt{3}} = -\dfrac{\pi}{6}$	MW1	
	(ii) 	MW3	
	(iii) $\angle AOC = \dfrac{\pi}{4} + \dfrac{\pi}{6}$	M1 W1	
	$= \dfrac{10\pi}{24}$		
	Hence $\angle AOB = \dfrac{5\pi}{24}$	MW1	
	$\Rightarrow \angle BOX = \dfrac{\pi}{4} - \dfrac{5\pi}{24}$		
	$= \dfrac{\pi}{24}$	MW1	
	$\Rightarrow \tan\dfrac{\pi}{24} = \dfrac{\sqrt{2}-1}{\sqrt{2}+\sqrt{3}}$	MW1	14
		Total	**75**