



Rewarding Learning

ADVANCED

General Certificate of Education

2016

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3



AMM31

[AMM31]

TUESDAY 21 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows two light elastic strings AP and PB used to keep a particle of weight 68 N in equilibrium at P. The ends A and B are attached to points 1.7 m apart on a fixed horizontal beam. AP is stretched to 0.8 m and BP is stretched to 1.5 m.

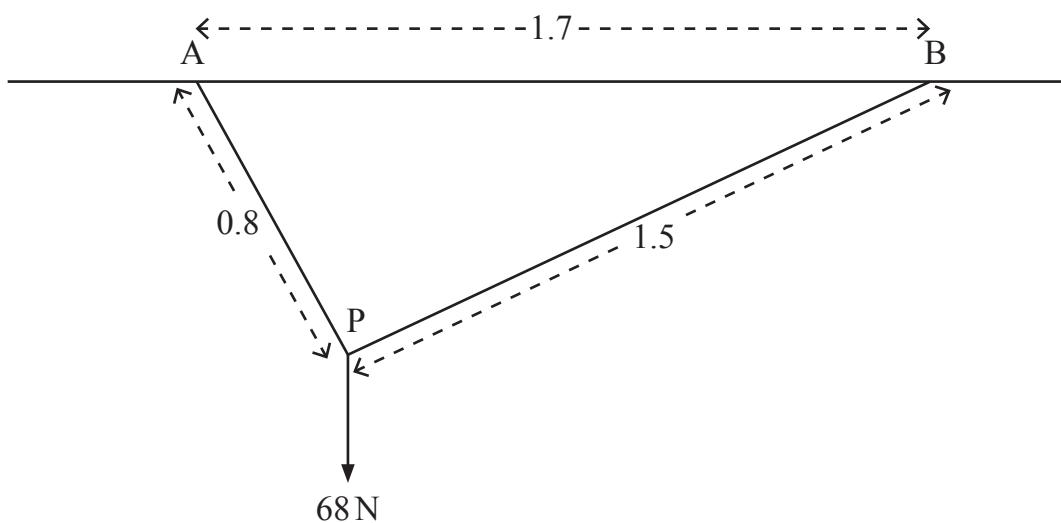


Fig. 1

(i) By resolving in the direction of AP, or otherwise, show that the tension in AP is 60 N. [4]

The extension of AP is 0.2 m.

(ii) Find the modulus of elasticity of AP. [2]

The stored elastic energy in BP is 8 J.

(iii) Find the extension in BP. [6]

2 A particle P is moving along the line whose vector equation is

$$\mathbf{r} = \lambda \begin{pmatrix} 1 \\ -2 \\ 4 \end{pmatrix}$$

under the action of two constant forces, \mathbf{F}_1 and \mathbf{F}_2 newtons where

$$\mathbf{F}_1 = \begin{pmatrix} -4 \\ 3 \\ -2 \end{pmatrix} \text{ and } \mathbf{F}_2 = \begin{pmatrix} 2k-4 \\ 4-k \\ k-9 \end{pmatrix}$$

A and B are two points on the line where λ takes the values 1 and -2 respectively. The distance AB is measured in metres.

(i) Show that the work done by \mathbf{F}_1 as P is moved from A to B is 54 J. [5]

The mass of P is 4 kg.

P is moving at 1 m s^{-1} at A. P is moving at 8 m s^{-1} at B.

(ii) Use the Work–Energy Principle to find k . [5]

3 The equation of motion of a particle moving in a straight line with S.H.M. of amplitude a is given by

$$\ddot{x} = -\omega^2 x$$

where x is the displacement from the centre of oscillation.

A particle P moving with S.H.M., of period 4π seconds, has velocity 1.4 m s^{-1} when x is 4.5 m .

(i) Find ω and a . [4]

(ii) Find the maximum speed of P during the motion. [1]

(iii) Find the maximum magnitude of the acceleration of P during the motion. [1]

(iv) Find the total time, in one complete oscillation, for which the speed of P is less than or equal to 1.4 m s^{-1} [5]

4 Fig. 2 below shows a design for a uniform metal logo ABCD in the shape of a trapezium with a rectangle PQRS removed.

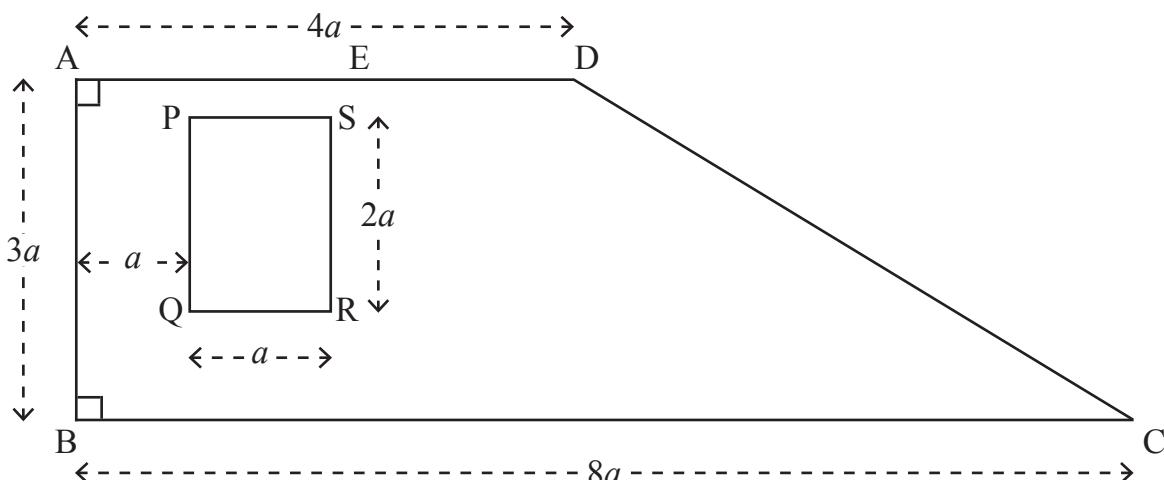


Fig. 2

$$\hat{A}\hat{B}C = \hat{D}\hat{A}B = 90^\circ$$

$$AB = 3a \text{ cm}, AD = 4a \text{ cm} \text{ and } BC = 8a \text{ cm}.$$

PQ is parallel to AB.

The distance between the parallel sides PQ and AB is a cm.

$$PQ = 2a \text{ cm and } QR = a \text{ cm.}$$

Model the logo as a lamina.

The centre of mass of the logo is at G.

(i) Show that the distance of G from AB is $\frac{53a}{16}$ cm.

[6]

E is the point on AD such that $AE = 2.5a$ cm.

The mass of the removed metal rectangle PQRS is m kg.

A small metal stud of mass M kg is attached at A.

(ii) When the logo is freely suspended from E, AD is horizontal. Find M in terms of m .

[4]

(iii) If, instead, the logo is freely suspended from A, find the minimum force in terms of m and g required to keep AD horizontal.

[3]

5 O is a fixed point on a straight horizontal line. A particle P of mass 6 kg is a distance x metres from O, where $x \geq 0$
A variable force F newtons acts on P in the direction OP. F is given by

$$F = 10 + 0.5e^{\frac{x}{2}} - 2x$$

(i) Find the acceleration of P at O. [2]

(ii) Show that the work done by F for $0 \leq x \leq 6$ is approximately 43.1 J [5]

The speed of P at O is 5 m s^{-1}

(iii) Find the speed of P when $x = 6$ [2]

(iv) Find x when F is a minimum. [6]

6 A ship B is 32 km away from a ship A on a bearing of 240° as shown in **Fig. 3** below.

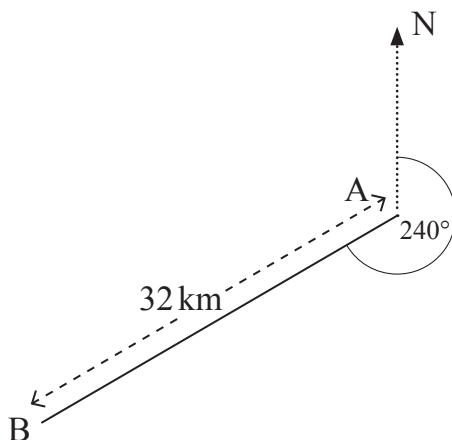


Fig. 3

Ship B is sailing due east at 16 km h^{-1}

A sets out to intercept B.

A travels at $v \text{ km h}^{-1}$ on a bearing of θ° as shown in **Fig. 4** below.

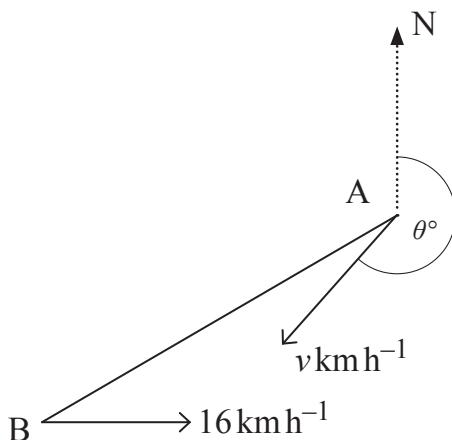


Fig. 4

(i) When $v = 24$, show that θ° is approximately 221°

[5]

(ii) Find the least value of v for which A can intercept B.

[1]

(iii) When $v = 12$, show, by drawing a velocity diagram, that A has a choice of two courses to intercept B.

Hence, find the greater time taken to intercept.

[8]

THIS IS THE END OF THE QUESTION PAPER
