Rewarding Learning

ADVANCED SUBSIDIARY (AS)
General Certificate of Education
2016

Mathematics

Assessment Unit F1
assessing
Module FP1: Further Pure Mathematics 1

AMF11

[AMF11]
MONDAY 27 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Let $\mathbf{A} = \begin{pmatrix} 5 & 4 \\ -3 & -2 \end{pmatrix}$ and $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

(i) Verify that $\mathbf{A}^2 = 3\mathbf{A} - 2\mathbf{I}$ [4]

(ii) Hence, or otherwise, express the matrix $\mathbf{A}^{-1}$ in the form $\alpha\mathbf{A} + \beta\mathbf{I}$, where α, β are real numbers. [3]

2 A system of linear equations is given by

$$\begin{aligned} 2x + (a-1)y - z &= 0 \\ (a+2)x + 3y &= 0 \\ 2x + 3y + (a+1)z &= 0 \end{aligned}$$

Find the values of a for which there are solutions other than $x = y = z = 0$ [5]

3 **(a)** The matrices **M**, **N** are given by $\mathbf{M} = \begin{pmatrix} 1 & 3 \\ 2 & -1 \end{pmatrix}$ and $\mathbf{N} = \begin{pmatrix} 0 & 4 \\ -2 & 1 \end{pmatrix}$

The matrix **S** represents the combined effect of the transformation represented by **M** followed by the transformation represented by **N**

(i) Find the matrix **S** [3]

A rectangle R is mapped to a new shape Q under the transformation represented by **S**

(ii) If the area of R is 3 cm^2, find the area of Q. [3]

(b) The matrix $\mathbf{P} = \begin{pmatrix} 3 & -1 \\ 0 & 1 \end{pmatrix}$ represents a linear transformation of the x–y plane.

Find the equation of the straight line through the origin, each of whose points is invariant under this transformation. [5]

4 The matrix $\mathbf{M} = \begin{pmatrix} 11 & 2 & 8 \\ 2 & 2 & -10 \\ 8 & -10 & 5 \end{pmatrix}$

(i) Given that $\begin{pmatrix} 1 \\ -2 \\ -2 \end{pmatrix}$ and $\begin{pmatrix} 2 \\ -1 \\ 2 \end{pmatrix}$ are eigenvectors of **M**, find the corresponding eigenvalues. [5]

(ii) Given that the third eigenvalue is 9, find a corresponding unit eigenvector. [6]

(iii) If **U** is a 3 × 3 matrix such that $\mathbf{U}^{\mathrm{T}}\mathbf{M}\mathbf{U} = \mathbf{D}$, where **D** is a diagonal matrix, write down a possible matrix **U** and the corresponding matrix **D** [3]

5 Two circles, C_1 and C_2, as shown in **Fig. 1** below, have a common chord, PQ, whose equation is $4x + 3y = 36$

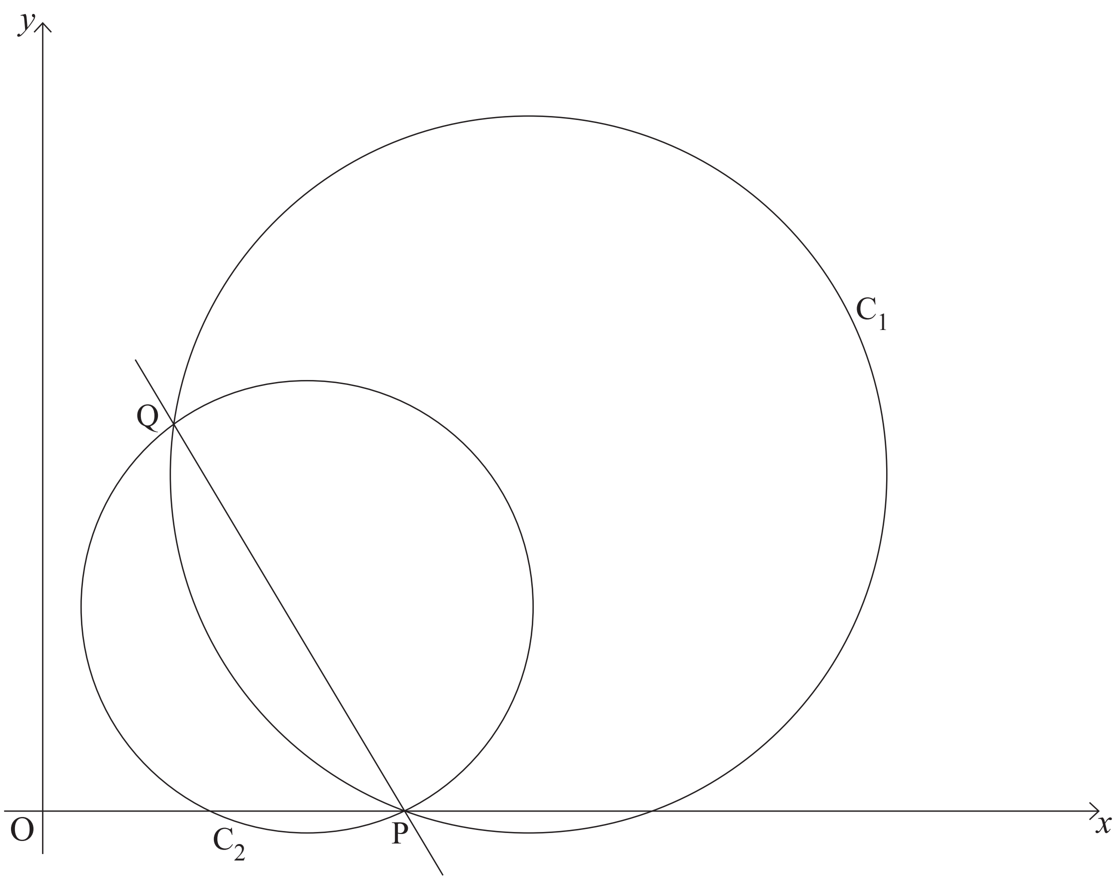

Fig. 1

(i) Given that the equation of circle C_1 is

$$x^2 + y^2 - 20x - 14y + 99 = 0$$

find the coordinates of P and Q. [6]

PQ is a diameter of the circle C_2

(ii) Show that the equation of C_2 is

$$x^2 + y^2 - 12x - 8y + 27 = 0$$ [4]

(iii) Find the equation of the tangent to circle C_2 at the point Q. [4]

6 G is the group of symmetries of an equilateral triangle, under composition of transformations. Its group table is

	a	b	c	d	e	f
a	c	d	a	b	f	e
b	f	e	b	a	c	d
c	a	b	c	d	e	f
d	e	f	d	c	a	b
e	d	c	e	f	b	a
f	b	a	f	e	d	c

(i) State the identity element. [1]

(ii) State whether the element a represents a reflection or a rotation. Justify your answer. [1]

(iii) Find a subgroup of order 3 [2]

The permutations

$$I = \begin{pmatrix} x & y & z \\ x & y & z \end{pmatrix} \qquad p = \begin{pmatrix} x & y & z \\ x & z & y \end{pmatrix} \qquad q = \begin{pmatrix} x & y & z \\ y & x & z \end{pmatrix}$$

$$r = \begin{pmatrix} x & y & z \\ z & y & x \end{pmatrix} \qquad s = \begin{pmatrix} x & y & z \\ z & x & y \end{pmatrix} \qquad t = \begin{pmatrix} x & y & z \\ y & z & x \end{pmatrix}$$

form a group H under composition.

(iv) Copy and complete the group table for H.

	I	p	q	r	s	t
I	I	p	q	r	s	t
p	p	I	t	s	r	q
q	q	s	I	t	p	r
r	r	t	s	I	q	p
s	s	q	r			
t	t	r	p			

[2]

(v) Find the period of the element s. [1]

(vi) State one element which is self-inverse. [1]

(vii) Show that groups G and H are isomorphic by stating clearly one possible isomorphism. [2]

7 The complex numbers z_1 and z_2 are given by $z_1 = \sqrt{2} + \sqrt{2}\,\mathrm{i}$ and $z_2 = \sqrt{3} - \mathrm{i}$

(i) Find the modulus and argument of each of z_1 and z_2 [6]

(ii) Plot the points representing each of z_1, z_2 and $z_1 + z_2$ on an Argand diagram. [3]

(iii) Hence find the exact value of $\tan \dfrac{\pi}{24}$ [5]

THIS IS THE END OF THE QUESTION PAPER