Rewarding Learning

ADVANCED
General Certificate of Education
2016

Mathematics

Assessment Unit M4
assessing
Module M4: Mechanics 4

AMM41

[AMM41]
WEDNESDAY 29 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A crane is modelled by four light smoothly jointed rods AD, BC, CD and BD. The framework is fixed to the horizontal ground at A and B and rests in a vertical plane. DC is horizontal and BD is vertical.
AB = $5l$, BD = $12l$ and CD = $9l$
A weight of 300 N acts at C, as shown in **Fig. 1** below.

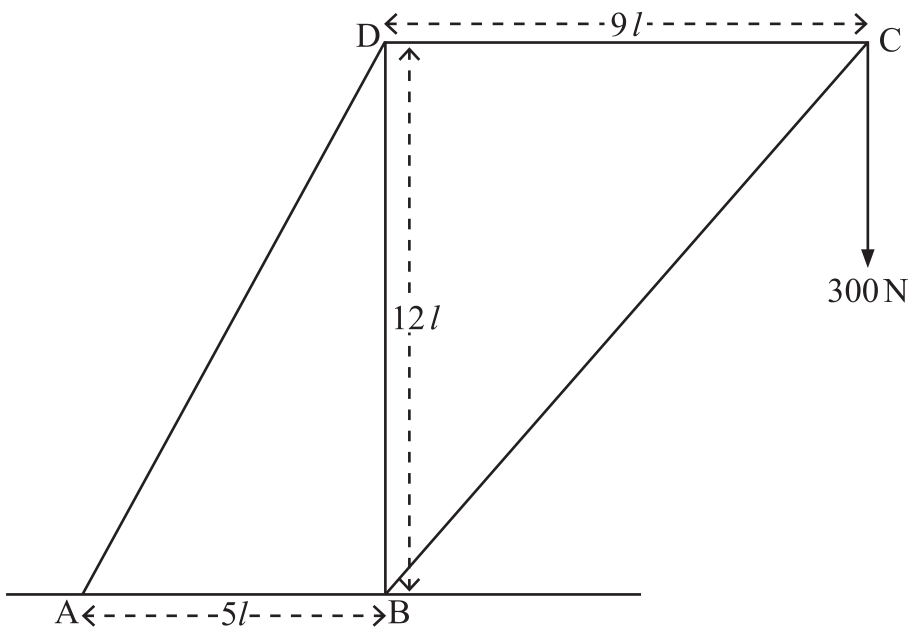

Fig. 1

(i) By considering the forces acting at C, find the forces in BC and CD, stating clearly whether each is a tension or thrust. [6]

(ii) Show that the magnitude of the force in the rod BD is 540 N. [3]

(iii) Find the magnitude of the reaction of the ground at B. [4]

2 A stuntman on a film set swings at one end of a rope and moves along an arc of a vertical circle.
The mass of the stuntman is 70 kg and the rope has length 12 m.
The stuntman can be modelled as a particle at one end of a light inextensible rope whose other end is fixed at C.
When the rope is vertical the stuntman is at A and has speed $5\,\mathrm{m\,s^{-1}}$
When the rope makes an angle of 25° with the downward vertical, the stuntman is at B and has speed $v\,\mathrm{m\,s^{-1}}$ as shown in **Fig. 2** below.

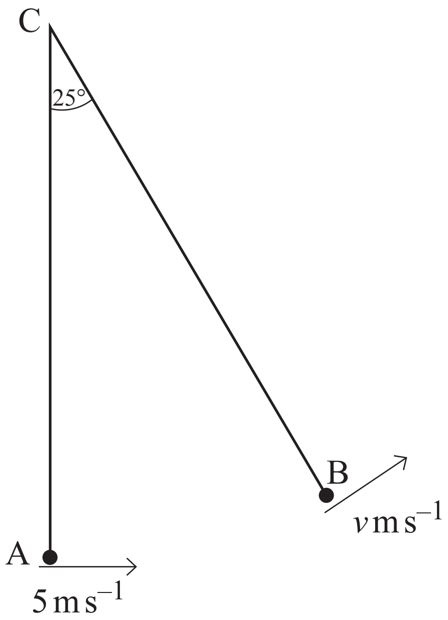

Fig. 2

Take the gravitational potential energy at A to be zero.

(i) Find v. [6]

(ii) Find the tension in the rope when the stuntman is at B. [4]

(iii) Find the maximum tension in the rope. [3]

3 A lamina of density $\rho\,\text{kg}\,\text{m}^{-2}$ can be modelled as the area bounded by the x-axis, the lines $x = 0$, $x = 8$ and the curve

$$y = (9 + 2x)^{\frac{1}{2}}$$

as shown in **Fig. 3** below.

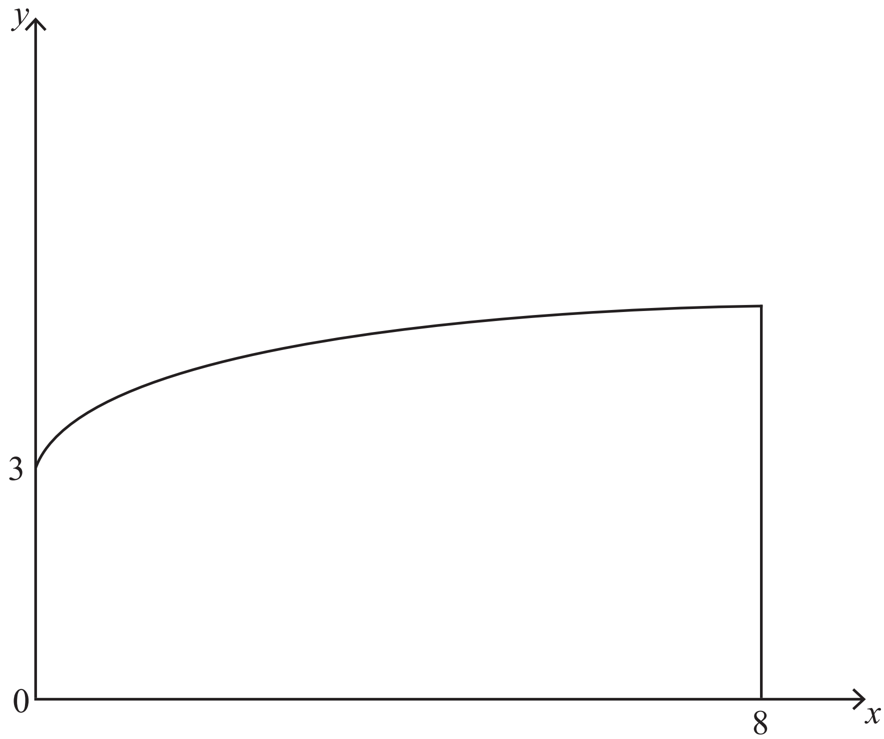

Fig. 3

(i) Show that the mass of the lamina is $\frac{98\rho}{3}$ kg [5]

The centre of mass of the lamina is at G.

(ii) Find the distance of G from the x-axis. [6]

4 A car of mass M kg is being tested on a road banked at an angle α to the horizontal. **Fig. 4** below represents the forces acting on the car with centre of mass G, h metres above the road and d metres from each side of the car.

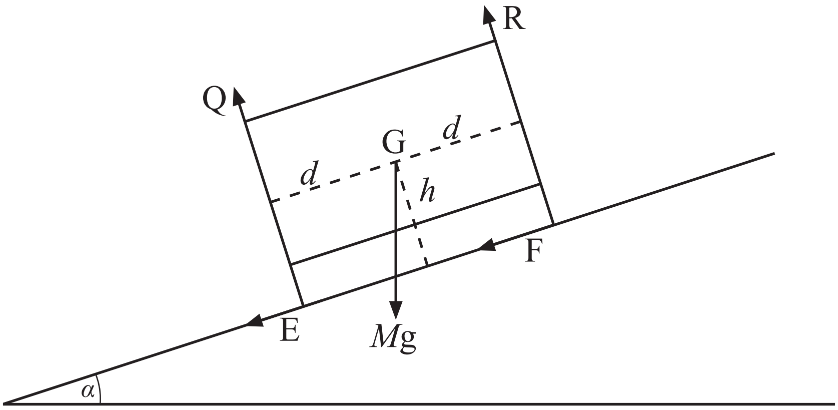

Fig. 4

When the car is travelling at v m s^{-1} on a road of circular radius r metres, it starts to topple outwards.

(i) Explain briefly why the force Q is zero at that instant. [1]

(ii) Show that

$$v^2 = rg\left(\frac{d + h\tan\alpha}{h - d\tan\alpha}\right)$$

[8]

5 **Fig. 5** below shows two balls A and B, of masses m and km respectively lying at rest on a smooth horizontal surface between two vertical walls P and Q.
A is given a horizontal velocity of $1\,\text{m}\,\text{s}^{-1}$ towards P and simultaneously B is given a horizontal velocity of $2\,\text{m}\,\text{s}^{-1}$ towards Q.

Fig. 5

A and B move in the same straight line perpendicular to P and Q. The coefficient of restitution between the balls and P and Q is e.

(i) Write down the velocities of A and B just after they collide with P and Q respectively. [2]

After impact with the walls, A and B collide directly with each other. The coefficient of restitution between A and B is also e.

(ii) Show that the velocity, V, of B just after this collision is

$$V = \frac{3e^2 + e(1 - 2k)}{1 + k}$$ [6]

(iii) Write down, in terms of k, the maximum value of V. [1]

As a result of this collision the direction of motion of A is reversed.

(iv) Show that

$$e > \frac{1 - 2k}{3k}$$ [6]

6 An observation satellite is placed in a circular orbit lying in a plane through the equator of the planet Jupiter.
The period, T, of the satellite is the same as the period of rotation of Jupiter so that the satellite remains above the same point on Jupiter's equator.
The mass of Jupiter is M.
Let r be the distance from the satellite to the centre of Jupiter.

(i) Show that

$$r = \left(\frac{GMT^2}{4\pi^2}\right)^{\frac{1}{3}}$$

where G is the universal gravitational constant. [5]

Given that G has dimensions $[M^{-1}L^3 T^{-2}]$,

(ii) show that this equation is dimensionally correct. [3]

Given that
$M = 1.90 \times 10^{27}$ kg
$G = 6.67 \times 10^{-11}$ m^3 kg^{-1}s^{-2}
$T = 3.564 \times 10^4$ s

(iii) find r. [2]

Given that the radius of Jupiter is 7.01×10^7 m,

(iv) find the minimum number of observation satellites in the same circular orbit of period T needed to view all points on the equator of Jupiter. [4]

THIS IS THE END OF THE QUESTION PAPER