Rewarding Learning

ADVANCED
General Certificate of Education
2017

Mathematics

Assessment Unit S4

assessing

Module S2: Statistics 2

[AMS41]

FRIDAY 23 JUNE, MORNING

MARK SCHEME

GCE ADVANCED/ADVANCED SUBSIDIARY (AS) MATHEMATICS

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right-hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for working.

MW indicates marks for combined method and working.

The solution to a question gains marks for correct method and marks for an accurate working based on this method. Where the method is not correct no marks can be given.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be followed through from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

Positive marking:

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier only a proportion of the marks will be available (based on the professional judgement of the examining team).

				AVAILABLE MARKS
1	**(i)**	$n = 8$ $\Sigma x = 577$ $\Sigma y = 42.9$ $\Sigma x^2 = 42245$ $\Sigma y^2 = 231.33$ $\Sigma xy = 3098$ $\quad r = \dfrac{Sxy}{\sqrt{Sxx\ Syy}}$	MW2	
		where	M1	
		$S_{xx} = 628.875$ $S_{yy} = 1.27875$ $S_{xy} = 3.8375$	W2	
		$r = \dfrac{3.8375}{\sqrt{628.875 \times 1.27875}} = 0.135$ (3 s.f.)	W1	
	(ii)	The low value of r suggests that there is little or no correlation between weight and lung capacity and that other factors are involved.	M2	8
2	**(i)**	$y = a + bx$		
		$b = \dfrac{S_{xy}}{S_{xx}} = \dfrac{\Sigma xy - \dfrac{\Sigma x \Sigma y}{n}}{\Sigma x^2 - \dfrac{(\Sigma x)^2}{n}}$	M1	
		$= \dfrac{457.25 - \dfrac{540 \times 7.683}{9}}{38400 - \dfrac{540^2}{9}}$	MW1	
		$= -6.22 \times 10^{-4}$ (3 s.f.)	W1	
		$a = \bar{y} - b\bar{x}$	M1	
		$= \dfrac{7.683}{9} - (-6.22 \times 10^{-4})\dfrac{540}{9}$		
		$= 0.891$ (3 s.f.)	W1	
		$\therefore y = 0.891 - (0.622 \times 10^{-4})x$ (3 s.f.)	W1	
	(ii)	$\hat{y} = 0.891 - (0.622 \times 10^{-4}) \times 65$	M1	
		$= 0.851$ (3 s.f.)	W1	8
3	**(i)**	Ten ‘suitable’ values	MW1	
	(ii)	Assign 2-digit value to each member. Take the first two digits of the random number set accepting those in range and ignoring duplicates until three have been chosen.	M5	6

		AVAILABLE MARKS
4 **(i)** $n = 50$ $\Sigma x = 1856.4$ $\Sigma x^2 = 68933.47$		
$\bar{x} = \dfrac{1856.4}{50} = 37.128$	MW1	
$\hat{\sigma}^2 = \dfrac{1}{49}\left(68933.47 - \dfrac{1856.4^2}{50}\right)$	M1	
$= 0.1847\ldots$	W1	
C.I. $= \bar{x} \pm 1.96 \times \dfrac{\hat{\sigma}}{\sqrt{n}}$	M1	
$= 37.128 \pm 1.96 \times \dfrac{\sqrt{0.1847\ldots}}{\sqrt{50}}$	MW2	
95% confidence interval is $(37.008\ldots, 37.247\ldots)$ $= (37.0, 37.2)$ (3 s.f.)	W2	
(ii) The probability that the interval contains the population mean is 0.95.	M1	9
5 $n = 40$ $\Sigma x = 1033.63$ $\Sigma x^2 = 26806.94$		
H_0: $\mu = 25.5$	M1	
H_1: $\mu > 25.5$	M1	
Carry out a one-tail upper z-test @ 5%	M1	
$z_{crit} = 1.645$	M1	
$\bar{x} = \dfrac{1033.63}{40} = 25.84075$	MW1	
$\hat{\sigma}^2 = \dfrac{1}{39}\left(26806.94 - \dfrac{1033.63^2}{40}\right) = 2.491\ldots$	MW1	
$z_{test} = \dfrac{\bar{x} - \mu}{\hat{\sigma}/\sqrt{n}}$		
$= \dfrac{25.84075 - 25.5}{\sqrt{2.491\ldots/40}} = 1.365\ldots$	MW1 W1	
Since $z_{test} < 1.645$ we do not reject H_0 and conclude that there is	M1	
insufficient evidence at 5% level to suggest that the average cost of a family visit to the cinema has increased.	M1	10

		AVAILABLE MARKS
6 From calculator		
$n = 12$ $\quad \Sigma x = 234.2$ $\quad \Sigma x^2 = 4574.96$		
H_0: $\mu = 20$	M1	
H_1: $\mu < 20$ (Prepared to accept $\mu \neq 20$)	M1	
Carrying out a t-test, lower tail, with 11 degrees of freedom.	M2	
$t_{crit} = -1.796$	M1	
$\bar{x} = \frac{234.2}{12} = 19.51\dot{6}$ $\quad \hat{\sigma}^2 = 0.37\dot{7}\dot{8}$	MW2	
$t_{test} = \dfrac{19.51\dot{6} - 20}{\sqrt{\dfrac{0.37\dot{7}\dot{8}}{12}}}$	MW1 W1	
$= -2.73$	W1	
Since $t_{test} < -1.796$ we reject H_0 and conclude that there is sufficient evidence at 5% to suggest that the makers' claim is not correct.	M1 M1	12
7 $X \sim N(3000, 60^2)$		
$\overline{X}_{10} \sim N\left(3000, \frac{60^2}{10}\right)$	M1	
$P(2980 < \overline{X}_{10} < 3020) = P\left(\frac{2980-3000}{60/\sqrt{10}} < Z < \frac{3020-3000}{60/\sqrt{10}}\right)$	M1	
$= P(-1.054 < Z < 1.054)$	W1	
$= 2\Phi(1.054) - 1$	M1	
$= 2 \times 0.8540 - 1$	W1	
$= 0.708$	W1	6

	AVAILABLE MARKS

8 **(a)** **(i)** $S \sim N(25, 10)$ $\quad T \sim N(40, 8)$

$P(S_1 + S_2 + S_3 > T_1 + T_2) = P(S_1 + S_2 + S_3 - T_1 - T_2 > 0)$ M1

$S_1 + S_2 + S_3 - T_1 - T_2 \sim N(25 + 25 + 25 - 40 - 40,\ 10 + 10 + 10 + 8 + 8)$

$\sim N(-5, 46)$ MW2

$P(S_1 + S_2 + S_3 - T_1 - T_2 > 0) = P\left(Z > \frac{0 - (-5)}{\sqrt{46}}\right)$

$= P(Z > 0.737)$ W1

$= 1 - 0.7694$
$= 0.2306$
$= 0.231$ (3 s.f.) MW1

(ii) $P(2S_1 - S_2 - 2T_1 + T_2 > 0)$

$2S_1 - S_2 - 2T_1 + T_2 \sim N(2 \times 25 - 25 - 2 \times 40 + 40,$
$4 \times 10 + 10 + 4 \times 8 + 8)$

$\sim N(-15, 90)$ MW3

$P(2S_1 - S_2 - 2T_1 + T_2 > 0) = P\left(Z > \frac{15}{\sqrt{90}}\right)$

$= P(Z > 1.581)$ W1

$= 1 - 0.9430$

$= 0.057$ W1

(b) $X \sim N(10, 4)$ $\quad Y \sim N(20, 3)$

$aX + bY \sim (10a + 20b, 4a^2 + 3b^2)$ MW2

$10a + 20b = 10 \rightarrow a + 2b = 1$ MW1

$4a^2 + 3b^2 = 112$

$\rightarrow 4(1 - 2b)^2 + 3b^2 = 112$

$19b^2 - 16b - 108 = 0$ W1

From quadratic formula

$b = 2.84\ldots$ or -2 W1

$a = 5 \qquad b = -2$ W1 16

Total **75**