Rewarding Learning

ADVANCED
General Certificate of Education
2017

Mathematics

Assessment Unit F3
assessing
Module FP3: Further Pure Mathematics 3

AMF31

[AMF31]
MONDAY 26 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$, where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find the angle between the planes

$$x - z = 23$$

and

$$x + y - 2z = 15$$ [4]

2 **(i)** Differentiate and simplify:

(a) $\tan^{-1}(\sinh x)$ [4]

(b) $\sin^{-1}(\tanh x)$ [4]

(ii) Hence express as simply as possible

$$\tan^{-1}(\sinh x) - \sin^{-1}(\tanh x)$$ [1]

3 The paths of submarines Adamant and Diamant are shown in **Fig. 1** below.

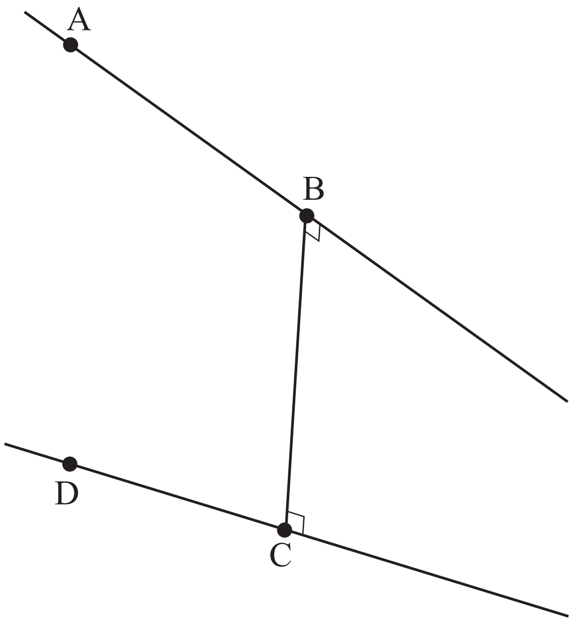

Fig. 1

The Adamant passes through the point A(1, 3, 2) and moves along the line

$$\mathbf{r_1} = \begin{pmatrix} 1 \\ 3 \\ 2 \end{pmatrix} + p \begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix}$$

and the Diamant passes through the point D (–4, –6, 7) and moves along the line

$$\mathbf{r_2} = \begin{pmatrix} -4 \\ -6 \\ 7 \end{pmatrix} + q \begin{pmatrix} -2 \\ -2 \\ 3 \end{pmatrix}$$

where the unit of length is the cable (0.1 nautical miles).
The shortest distance between their paths is BC.

(i) Find the unit vector $\hat{\mathbf{n}}$ in the direction of $\overrightarrow{BC}$ [3]

(ii) By writing $\overrightarrow{AD} = \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CD}$ and evaluating $\overrightarrow{AD} \cdot \hat{\mathbf{n}}$, find the distance BC. [4]

4 **(i)** Prove that

$$\tanh^{-1} x \equiv \frac{1}{2} \ln\left(\frac{1+x}{1-x}\right)$$ [4]

(ii) By using integration by parts, and without fully evaluating either integral, show that

$$\int_k^{\frac{(1-k)}{(1+k)}} \frac{\ln\left(\frac{1}{x}\right)}{1-x^2} \, dx = \int_k^{\frac{(1-k)}{(1+k)}} \frac{\tanh^{-1} x}{x} \, dx$$

where $0 < k < \sqrt{2} - 1$ [7]

5 Tetra-Tents are designing a new model as shown in **Fig. 2** below.

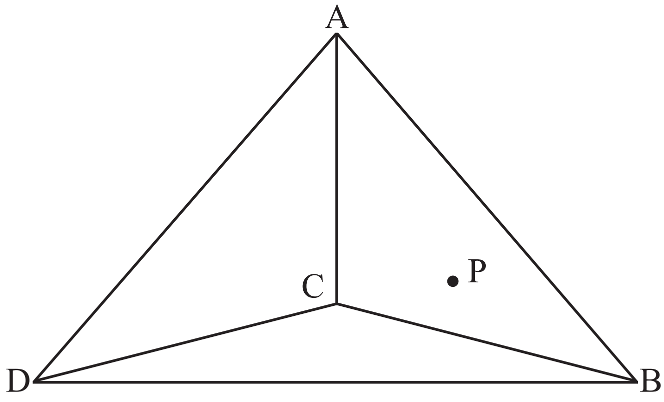

Fig. 2

They intend to attach a doorbell at position P.

The plane ACD has equation $\mathbf{r} \cdot \begin{pmatrix} 2 \\ -3 \\ -1 \end{pmatrix} = 3$

The line AB has equation $\dfrac{x-1}{-1} = \dfrac{y+1}{1} = \dfrac{z-3}{-4}$

The point P $\left(2\frac{1}{3}, -\frac{2}{3}, 4\right)$ lies in the plane ABC.

(i) Find the coordinates of the apex, A. [4]

(ii) Find an equation of the line AC. [10]

6 **(i)** Using the exponential definitions of sinh x and cosh x, prove the identity

$$\sinh 2x \equiv 2 \sinh x \cosh x$$ [3]

(ii) Using the substitution $x + 2 = 3 \cosh u$, prove that

$$\int \sqrt{(x+5)(x-1)}\, dx = \frac{1}{2}(x+2)\sqrt{x^2+4x-5} - \frac{9}{2}\ln\left[(x+2)+\sqrt{x^2+4x-5}\right] + c$$ [10]

7 The integral I_n is defined as

$$I_n = \int \frac{x^n}{\sqrt{a^2 - x^2}}\, dx$$

where $n \geqslant 0$

(i) Derive the reduction formula,

$$n\, I_n = -x^{n-1}\sqrt{a^2 - x^2} + a^2(n-1)\, I_{n-2}$$

where $n \geqslant 2$ [8]

(ii) Hence find

$$\int \frac{(x^3 + 3x^2 + 3x + 7)}{\sqrt{15 - 2x - x^2}}\, dx$$ [9]

THIS IS THE END OF THE QUESTION PAPER