Rewarding Learning

ADVANCED

General Certificate of Education

2017

Mathematics

AMM31

Assessment Unit M3

assessing

Module M3: Mechanics 3

[AMM31]

TUESDAY 20 JUNE, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 A particle P of mass m kg rests on a rough plane inclined at 60° to the horizontal.
The coefficient of friction between P and the plane is μ.
A is a fixed point at the top of the plane, 0.6 m above P along a line of greatest slope.
P is attached to A by a light elastic string of natural length of 0.4 m and modulus of elasticity $\frac{2\sqrt{3}}{3}mg$ N.
P is about to slide down the plane.

(i) Draw a diagram showing all the external forces acting on P. [2]

(ii) Find the value of μ. [8]

2 A particle P of mass $\frac{7}{3}$ kg is initially at rest at the point A whose position vector is $(2\mathbf{i} + \mathbf{j} + 6\mathbf{k})$ m.
P is acted on by a force

$$\mathbf{F} = \left[(\alpha + \beta)\mathbf{i} + (2 + 3\alpha)\mathbf{j} + (-3 + 2\beta)\mathbf{k}\right] \text{N}$$

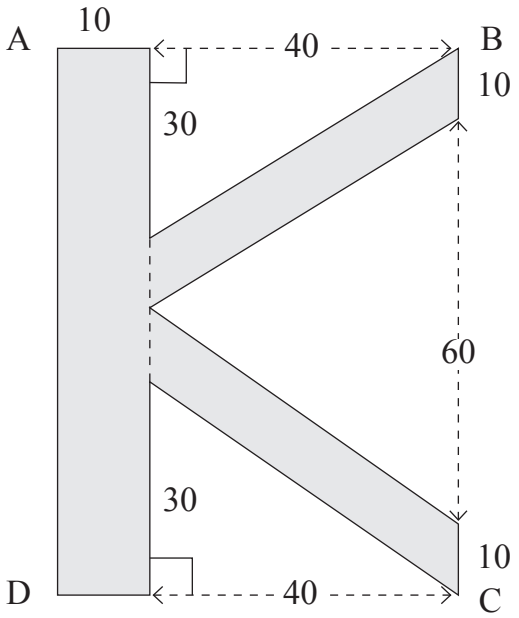
as it moves from A to the point B.
The position vector of B is $(8\mathbf{i} - 2\mathbf{j} + 15\mathbf{k})$ m.

(i) Given that **F** acts parallel to **AB**, find the values of α and β. [7]

(ii) Calculate the work done by **F**. [2]

(iii) Hence find the speed of P when it reaches B. [2]

3 The letter "K" in an advertising hoarding is made from a rectangle ABCD of uniform thin sheet metal with three triangles removed, as shown in **Fig. 1** below.



(All dimensions are in centimetres)

Fig. 1

(i) Show that the centre of mass of the letter is 17.5 cm from the edge AD. [7]

The letter is suspended in equilibrium by vertical forces P and Q acting at A and B respectively.

(ii) Given that AD is vertical, find Q in terms of P. [6]

4 A mass of 3 kg, suspended from a fixed point K by a light elastic string, hangs in equilibrium a distance $5d$ vertically below K.
When the 3 kg mass is replaced by a mass of 2 kg, this mass hangs in equilibrium a distance $4d$ vertically below K.
Model the masses as particles.

(i) Find the natural length l, in terms of d, and find the modulus of elasticity of the elastic string. [7]

The mass of 2 kg is now held at K and released from rest. It comes to instantaneous rest a distance h metres below K, where $h > 2d$.

(ii) Show that

$$h^2 - 8hd + 4d^2 = 0$$ [5]

5 A coastguard vessel C sights a boat B 25 km away on a bearing of 220°
B is sailing at 15 kmh^{-1} on a bearing of 280° as shown in **Fig. 2** below.

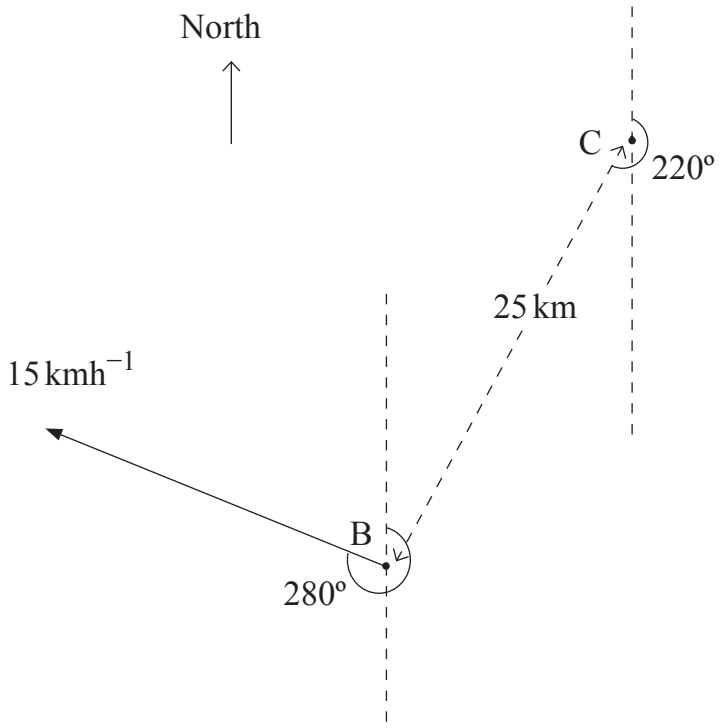


Fig 2

C is sailing at 40 kmh^{-1}

(i) Find the course C should steer in order to intercept B. [6]

(ii) Find the time it will take for C to intercept B. [4]

6 A particle P moves in a straight line with simple harmonic motion about a fixed point O. When P passes through the point A, where OA = 1 m, it has a speed of $3\sqrt{3}\,\text{ms}^{-1}$ and an acceleration of magnitude $9\,\text{ms}^{-2}$

(i) Find the period and amplitude of the simple harmonic motion. [7]

P passes through A travelling towards O.

(ii) Find the time taken for P to return to A for the first time. [4]

7 A simple pendulum consists of a particle suspended from a fixed point P by a light inextensible string of length 0.4 m.
The period, T, of small oscillations of a simple pendulum is given by $T = 2\pi\sqrt{\frac{l}{g}}$

where l is the length of the pendulum and g is the acceleration due to gravity.

(i) Find the period of the oscillations of this simple pendulum at a point where the acceleration due to gravity is $9.8\ \text{ms}^{-2}$ [1]

(ii) Hence find the number of oscillations, to the nearest whole number, this pendulum will make in one day. [1]

The pendulum is now taken to a place where it makes 500 fewer oscillations per day. Its length is adjusted until it again makes the original number of oscillations per day.

(iii) Find the new length of the pendulum. [6]

THIS IS THE END OF THE QUESTION PAPER