Rewarding Learning

ADVANCED

General Certificate of Education

2017

Mathematics

AMM41

Assessment Unit M4

assessing

Module M4: Mechanics 4

[AMM41]

WEDNESDAY 28 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all seven** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{ms}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all seven questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Forces of magnitude 6 N, 6 N, 9 N, P N and Q N act along the sides AB, BC, CD, DA and the diagonal AC of the rectangle ABCD as shown in **Fig. 1** below.

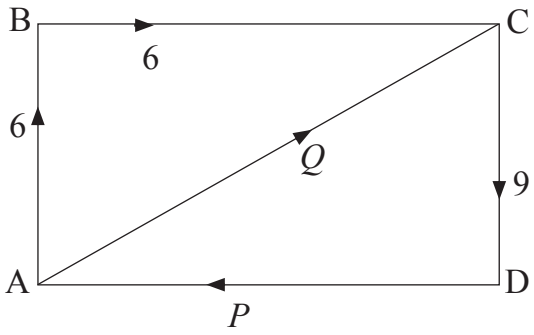

Fig. 1

AB = 3 m, BC = 4 m

Given that the resultant force of this system is zero:

(i) find P and Q; [6]

(ii) find the resultant moment of the system. [3]

2 A paperweight can be modelled as a right circular uniform solid cone of base radius r and height $2l$ joined to a uniform solid cylinder of radius r and height l at their circular faces, as shown in **Fig. 2** below.

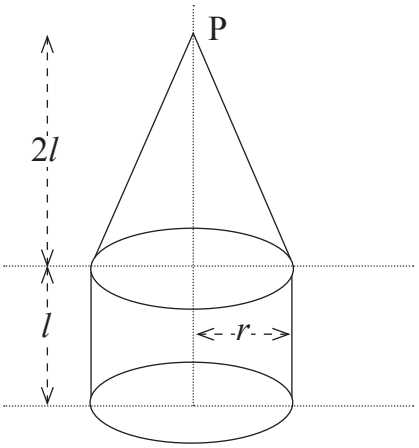

Fig. 2

The cone and the cylinder are made from the same material.

(i) State why the centre of mass of the cone is $\frac{l}{2}$ vertically above the centre of its circular base. [1]

(ii) Given that the mass of the cone is M, find the total mass of the paperweight in terms of M. [3]

The paperweight is placed on a rough horizontal surface with its axis of symmetry vertical.
A horizontal force F is applied at its vertex P.
The surface is rough enough to prevent sliding.

(iii) Given that the paperweight is about to topple, find F in terms of M, r, g and l. [3]

3 When a satellite of mass m is at rest on the surface of the Earth its weight is mg.

(i) Given that the mass of the Earth is M and its radius is R, obtain an expression for G, the Universal Gravitational Constant, in terms of M, R and g. [2]

(ii) Hence find the dimensions of G. [3]

The satellite is placed in a circular orbit around the Earth, travelling at 7500 ms^{-1}

(iii) Given that $R = 6.44 \times 10^6$ m, find the height of this orbit above the Earth's surface. [6]

4 **Fig. 3** below shows a ship's derrick modelled as a light pin-jointed framework ACBD. The derrick is freely hinged to a fixed vertical mast at A and carries a load of weight 6000 N at B.
The derrick is held in equilibrium by the tension T in a horizontal cable joining B to the point P in the mast vertically above A.

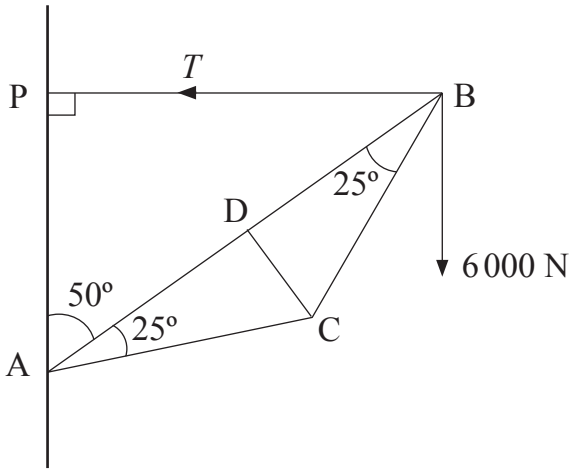

Fig. 3

AD = DB = 6 m

(i) Find T. [3]

(ii) Find the magnitudes of the forces in BC and BD. [6]

5 A small smooth bead P of mass m is threaded on to a smooth wire in the shape of a circle, centre C and radius r.
The wire is fixed in a vertical plane as shown in **Fig. 4** below.

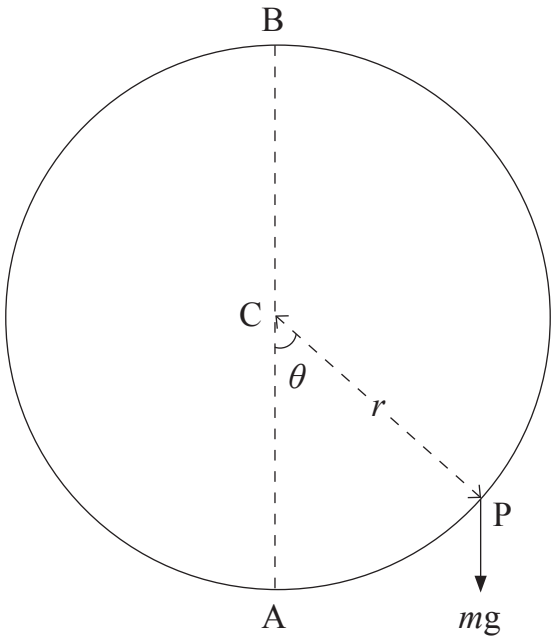

Fig. 4

AB is the vertical diameter of the circle.
P is projected with speed u from A.
When CP makes an angle θ with the downward vertical, the speed of the bead is v.

(i) Obtain an expression for v^2 in terms of u, θ, g and r. [4]

(ii) Hence find u_{min} the minimum value of u for which P will reach B. [2]

(iii) If $u = 2u_{min}$ find the magnitude of the reaction between P and the wire when P reaches B. [6]

6 A van of mass m travels around a bend in a road. The bend is a circular arc of radius r and is banked at an angle θ to the horizontal. When the van travels round the bend with speed u it is on the point of slipping down the banking.
The coefficient of friction between the van's tyres and the road surface is μ.
Model the van as a particle.

(i) Draw a diagram showing all the external forces acting on the van. [2]

(ii) Obtain an expression for $\tan\theta$ in terms of u, g, r and μ. [10]

(iii) Given that $u = 15\,\text{ms}^{-1}$, $r = 100\,\text{m}$ and $\mu = 0.4$, find θ. [2]

(iv) Explain briefly why this is not a suitable road design. [1]

7 A and B are small smooth spheres of equal radii and masses m and $4m$ respectively.
They lie at rest on a smooth horizontal surface.
A is projected with speed u and collides directly with B.
The coefficient of restitution between A and B is e.

(i) Find, in terms of m, u and e, the velocities of A and B after the collision. [6]

The direction of motion of A is reversed by the collision.

(ii) Show that $e > \frac{1}{4}$ [2]

B now collides with a fixed vertical surface perpendicular to its direction of motion.
The coefficient of restitution between B and the surface is also e.

(iii) Write down the velocity of B after it collides with the surface. [1]

(iv) Find the maximum value of e such that B will collide with A again. [3]

THIS IS THE END OF THE QUESTION PAPER