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General Certificate of Education

2018

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# Mathematics

Assessment Unit F2  
*assessing*  
Module FP2: Further Pure Mathematics 2



AMF21

**[AMF21]**  
**TUESDAY 19 JUNE, MORNING**

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## TIME

1 hour 30 minutes.

## INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all eight** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

## INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is  $\ln z$  where it is noted that  $\ln z \equiv \log_e z$

**Answer all eight questions.**

**Show clearly the development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Find, in radians, the general solution of the equation

$$\sin 2\theta = 1 + \cos 2\theta \quad [6]$$

**2 (i)** Show that for  $r \geq 1$

$$\frac{1}{r(r+1)} - \frac{1}{(r+1)(r+2)} = \frac{2}{r(r+1)(r+2)} \quad [1]$$

**(ii)** Hence or otherwise show that

$$\sum_{r=1}^n \frac{1}{r(r+1)(r+2)} = \frac{n(n+3)}{4(n+1)(n+2)} \quad [3]$$

**(iii)** Using **(ii)** evaluate

$$\sum_{r=1}^{\infty} \frac{1}{r(r+1)(r+2)} \quad [1]$$

**3** Using partial fractions, show that

$$\int_0^1 \frac{x+3}{(x+1)(x^2+4x+5)} dx = \frac{1}{2} \ln 2 \quad [10]$$

**4 (i)** Given that

$$f(x) = e^{-mx} - (1+2x)^{-n}$$

find the Maclaurin expansion for  $f(x)$  up to and including the term in  $x^2$  [6]

**(ii)** Given that the first non-zero term in this expansion is  $-4x^2$ , find the values of  $m$  and  $n$ . [3]

**5** The equation of the tangent to the ellipse

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

at any point  $(a \cos \theta, b \sin \theta)$  on the ellipse is given by

$$\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$$

**(i)** Tangents are drawn to this ellipse at the points  $(a \cos \alpha, b \sin \alpha)$  and  $(a \cos \beta, b \sin \beta)$ .  
The tangents are perpendicular.

Show that

$$\tan \alpha \tan \beta = \frac{-b^2}{a^2} \quad [3]$$

**(ii)** A line joins P  $(a \cos \theta, b \sin \theta)$  to the fixed point S  $(a, 0)$  on this ellipse. As  $\theta$  varies,  
find the cartesian equation of the locus of the midpoint of the chord PS. [6]

**6** Using the principle of mathematical induction, prove that for  $n \geq 0$

$$5^n + 11^{n+1} \text{ is divisible by } 6 \quad [6]$$

7 (i) Given that

$$(\cos \theta + i \sin \theta)^n \equiv \cos n\theta + i \sin n\theta$$

when  $n$  is a positive integer, deduce that the statement is also true when  $n$  is a negative integer.

[4]

(ii) Using (i), show that if  $Z = \cos \theta + i \sin \theta$ , then

$$Z^n + Z^{-n} = 2 \cos n\theta \quad [2]$$

(iii) By considering  $(Z + Z^{-1})^4$ , show that

$$\cos^4 \theta = \frac{1}{8} (\cos 4\theta + 4 \cos 2\theta + 3) \quad [3]$$

(iv) Hence or otherwise find the exact value of

$$\int_0^{\frac{\pi}{6}} \cos^4 \theta \, d\theta \quad [5]$$

8 (i) Find the general solution of the differential equation

$$\lambda \frac{d^2y}{dx^2} + (\lambda^2 + 1) \frac{dy}{dx} + \lambda y = 0$$

(a) when  $\lambda \neq 0$  and  $\lambda \neq 1$  [4]

(b) when  $\lambda = 1$  [2]

(ii) Hence or otherwise determine the solution of

$$2 \frac{d^2y}{dx^2} + 5 \frac{dy}{dx} + 2y = 2x^2 - 1$$

where  $y = 9$  when  $x = 0$  and  $\frac{dy}{dx} = 0$  when  $x = 0$  [10]

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**THIS IS THE END OF THE QUESTION PAPER**

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