Rewarding Learning

**ADVANCED**

**General Certificate of Education**

**2018**

# Mathematics

Assessment Unit F3

*assessing*

Module FP3: Further Pure Mathematics 3

AMF31

**[AMF31]**

**THURSDAY 21 JUNE, MORNING**

**TIME**

1 hour 30 minutes.

**INSTRUCTIONS TO CANDIDATES**

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

**INFORMATION FOR CANDIDATES**

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$, where it is noted that $\ln z \equiv \log_e z$

**Answer all eight questions.**

**Show clearly the full development of your answers.**

**Answers should be given to three significant figures unless otherwise stated.**

**1** Vectors **a**, **b** and **c** are given by:

$$\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}$$

$$\mathbf{b} = 3\mathbf{i} - \mathbf{j} + 2\mathbf{k}$$

$$\mathbf{c} = 4\mathbf{i} + 2\mathbf{j} - 5\mathbf{k}$$

Evaluate $\mathbf{a} . \mathbf{b} \times \mathbf{c}$ [4]

**2** **(i)** Show that

$$\sinh^{-1} x \equiv \ln(x + \sqrt{x^2 + 1})$$ [4]

**(ii)** Find the exact solutions of

$$\cosh^2 x = 9 + 2 \sinh x$$

giving your answers in logarithmic form. [5]

**3** **(a)** Find a vector equation of the line common to the planes

$$2x - 3y + z = 4$$

and $x - 2y + 4z = 6$ [6]

**(b)** Show that the lines

$$\mathbf{r} = (-2\mathbf{i} - \mathbf{j} + 7\mathbf{k}) + \lambda(\mathbf{i} + 5\mathbf{j} + 3\mathbf{k})$$

and $\mathbf{r} = (6\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}) + \mu(-5\mathbf{i} + \mathbf{j} - 2\mathbf{k})$

have no common point. [6]

**4** Differentiate

$$\tan^{-1}(2x^2+3)+\cos^{-1}\frac{x}{x+1}$$ [7]

**5** Let

$$I_n = \int \tan^n x \, dx \qquad n \geqslant 0$$

**(i)** Show that

$$I_n = \frac{1}{n-1}\tan^{n-1}x - I_{n-2} \qquad n \geqslant 2$$ [5]

**(ii)** Hence find the exact value of

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \tan^5 x \, dx$$ [5]

**6** Show that

$$\int e^x \sin x \, dx = Ae^x \sin(x+B)+c$$

where $A$ and $B$ are constants to be determined. [8]

**7** An arrow is embedded in an archery target. These can be modelled by a line and a plane. The line has equation

$$\frac{x - 4.2}{3} = \frac{y - 3}{1} = \frac{z - 1.1}{-2}$$

The plane has equation

$$7x + y - 4z = 25$$

**(i)** Find the coordinates of the point of intersection of the line and the plane. [5]

**(ii)** Find the angle between the line and the plane. [6]

**8** **Fig. 1** below shows sketches of the graphs of $y = \tanh x$ and $y = \text{sech}\, x$

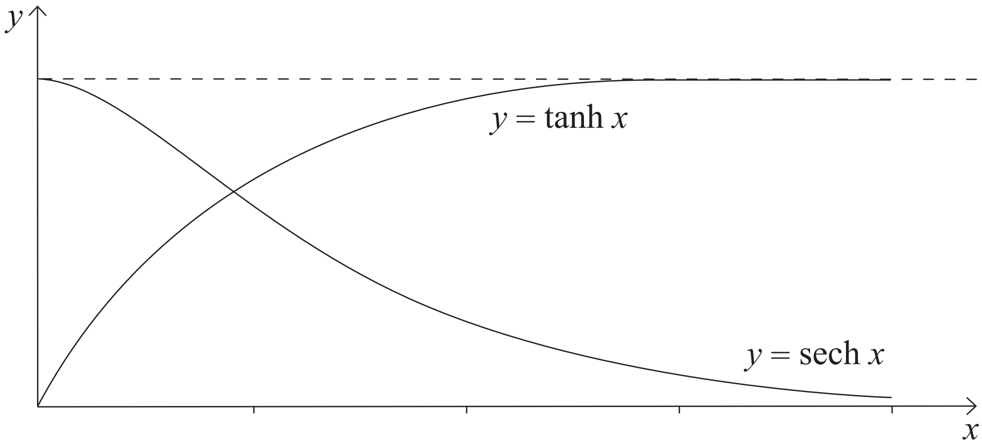


**Fig. 1**

**(i)** Prove that the graphs of $y = \tanh x$ and $y = \text{sech}\, x$ intersect on the line $x = \ln(1 + \sqrt{2})$ [4]

**(ii)** The behaviour of a stock market commodity can be modelled by the exact area $A$ between the $x$-axis and the curve $\text{q}(x)$ defined by

$$\text{q}(x) = \begin{cases} \tanh x & 0 \leqslant x \leqslant \ln(1+\sqrt{2}) \\ \text{sech}\, x & \ln(1+\sqrt{2}) \leqslant x \leqslant \infty \end{cases}$$

Find $A$. [10]

**THIS IS THE END OF THE QUESTION PAPER**