Rewarding Learning

ADVANCED

General Certificate of Education

2018

Mathematics

Assessment Unit M4

assessing

Module M4: Mechanics 4

AMM41

[AMM41]

MONDAY 25 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
Answers should include diagrams where appropriate and marks may be awarded for them.
Take $g = 9.8\,\text{m}\,\text{s}^{-2}$, unless specified otherwise.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 **Fig. 1** below shows a crane ABCDE.
AB = BC = CD = DE = DB = BE = AE = 4 m.
The crane is smoothly hinged at A to a fixed point.
The crane is maintained in equilibrium in the vertical plane with DE horizontal by a horizontal force F newtons applied at E in the direction $\overrightarrow{DE}$.
A load of 3000 N is suspended at C.

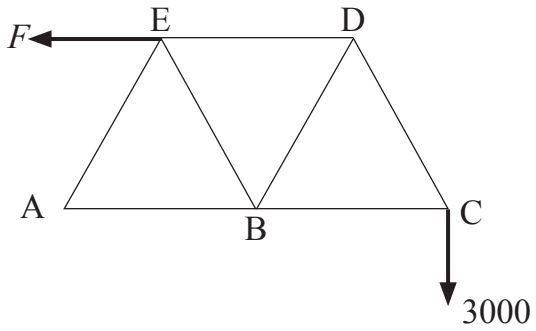

Fig. 1

Model the crane as a light pin-jointed framework.

(i) Find F. [3]

(ii) Find the magnitude of the reaction at A. [5]

(iii) Find the force in DE, stating clearly if it is a tension or a thrust. [7]

2 **Fig. 2** below shows a uniform lamina ABCD in the shape of the area enclosed between the curve $y = \frac{4}{x^2}$, the x-axis and the lines $x = 1$ and $x = 2$

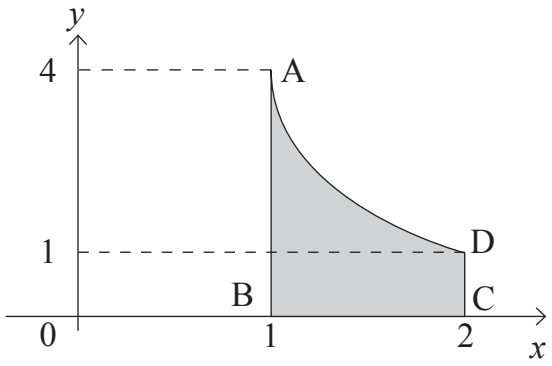

Fig. 2

All dimensions are in metres.
The material from which the lamina is made has a density of $1\,\text{kg}\,\text{m}^{-2}$
The area of the lamina is $2\,\text{m}^2$
The centre of mass of the lamina is at the point (X, Y).

(i) Show that $Y = \frac{7}{6}$ [7]

(ii) Find X. [3]

The lamina is freely suspended from A.

(iii) Find the angle the edge AB makes with the vertical. [4]

3 A light rigid rod AB of length r metres is freely hinged at A to a fixed point and has a particle of mass m kg fastened at B. The rod is free to rotate about A in the vertical plane. While hanging in equilibrium vertically below A, the particle is given a horizontal speed $u\,\mathrm{m\,s^{-1}}$

(i) Show that for the particle to perform a full circle about A,

$$u > 2\sqrt{gr}$$

where g is the acceleration due to gravity. [6]

(ii) If the tension in the rod is zero when the particle is vertically above A, find u. [5]

4 A liquid with density $\rho\,\mathrm{kg\,m^{-3}}$ flows with velocity $v\,\mathrm{m\,s^{-1}}$ through a pipe of diameter D metres. The pressure drop $P\,\mathrm{N\,m^{-2}}$ between two points a distance L metres apart is proportional to ρ, v, D and L.

(i) Use the Method of Dimensions to show that a possible formula for P is

$$P = k\left(\frac{L}{D}\right)^{\alpha}\rho v^2$$

where k and α are arbitrary constants. [8]

When the same liquid flows through a pipe of diameter $\frac{D}{2}$ metres with velocity $2v\,\mathrm{m\,s^{-1}}$ the pressure drop between two points a distance L metres apart is 8 times as great.

(ii) Find α. [3]

5 **Fig. 3** below shows a rectangle ABCD where AB = 10 m and AD = 6 m.
E is the point on AB such that $\hat{ADE} = 50°$

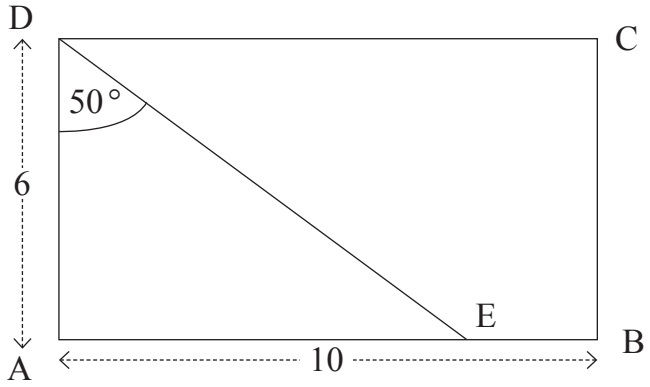

Fig. 3

Forces of 2 N, 4 N, 1 N, 8 N and 10 N act along $\overrightarrow{AD}$, $\overrightarrow{CD}$, $\overrightarrow{BC}$, $\overrightarrow{BA}$ and $\overrightarrow{DE}$ respectively.
When a force R, acting at A, is added to this system of forces it reduces the system to a couple.

(i) Find the magnitude and direction of R. [8]

(ii) Find the magnitude of the couple. [3]

6 A and B are small smooth spheres of equal radii and masses m and $3m$ respectively. They lie in a straight smooth horizontal groove which meets a smooth vertical wall at right angles.
Initially the spheres are at rest with B between A and the wall.
The coefficient of restitution between A and B and between B and the wall is e.
A is propelled towards B with speed u.
When A collides with B its direction of motion is reversed.

(i) Show that $e > \frac{1}{3}$ [6]

(ii) Find, in terms of u and e, the speed of B when it rebounds from the wall. [3]

(iii) Find the range of values of e for which B will collide with A again. [4]

THIS IS THE END OF THE QUESTION PAPER