Rewarding Learning

ADVANCED SUBSIDIARY (AS)

General Certificate of Education

2018

Mathematics

Assessment Unit F1

assessing

Module FP1: Further Pure Mathematics 1

AMF11

[AMF11]

TUESDAY 22 MAY, AFTERNOON

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all six** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 The matrix $\mathbf{M} = \begin{pmatrix} 2 & p \\ 1 & 5 \end{pmatrix}$

(i) Given that one eigenvalue of **M** is 6, calculate the value of p. [4]

(ii) Find the other eigenvalue of **M.** [3]

(iii) For the eigenvalue 6, find a corresponding unit eigenvector. [4]

2 **(i)** Copy and complete the table, given in **Fig. 1** below, for the group G_1 formed under multiplication modulo 15

$\times_{15}$	3	6	9	12
3	9	3	12	6
6	3			
9	12			
12	6			

[3]

Fig. 1

A logo consists of 4 congruent equally spaced shapes as shown in **Fig. 2** below.

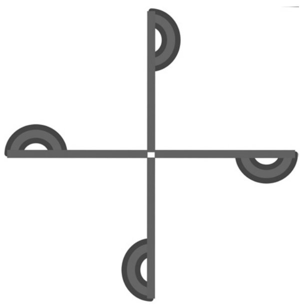

Fig. 2

(ii) Define clearly the symmetries of this logo. [2]

(iii) Hence construct the table for the symmetry group G_2 of this logo. [4]

(iv) Write down an isomorphism between G_1 and G_2 [2]

The group G_3 has the table given in **Fig. 3** below.

	e	a	b	c
e	e	a	b	c
a	a	e	c	b
b	b	c	e	a
c	c	b	a	e

Fig. 3

(v) Explain why G_1 and G_3 are not isomorphic. [1]

3 The equation of a circle is given by

$$x^2 + y^2 - 2x - 14y + 45 = 0$$

(i) Find the equation of the tangent to this circle at the point (2, 9). [5]

(ii) Verify that the point (6, 7) lies on this tangent. [1]

(iii) Hence, or otherwise, find the equation of the other tangent from (6, 7) to this circle. [4]

4 **(a)** A shear of the x–y plane maps the points (1, 1) and (3, 2) to the points (−1, −5) and (−4, −19) respectively.

(i) Find the matrix which defines this transformation. [5]

The transformation has an invariant line which passes through the origin.

(ii) Find the equation of this invariant line. [5]

(b) **(i)** Describe the transformation given by the matrix

$$\mathbf{A} = \begin{pmatrix} -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{\sqrt{3}}{2} & -\frac{1}{2} \end{pmatrix}$$ [2]

(ii) Find the smallest positive integer n such that $\mathbf{A}^n = \mathbf{I}$, where $\mathbf{I}$ is the identity matrix. [2]

5 **(a)** Express the complex number

$$\frac{3 + 4\mathrm{i}}{1 - 7\mathrm{i}}$$

in the form $a + b\mathrm{i}$, where a and b are real numbers. [3]

(b) **(i)** Sketch on an Argand diagram the locus of those points z which satisfy

$$|z - (3 + 7\mathrm{i})| = 6$$ [3]

(ii) On the same diagram, sketch the locus of those points w which satisfy

$$|w - (3 + 7\mathrm{i})| = |w - (-9 - 9\mathrm{i})|$$ [3]

(iii) Hence, or otherwise, find the minimum value of $|z - w|$, where z, w are complex numbers which satisfy the equations in **(i)** and **(ii)** respectively. [3]

6 **(a)** $\mathbf{A} = \begin{pmatrix} 2 & 3 & 4 \\ 1 & 7 & -2 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 0 \\ -1 & 3 \\ 0 & 4 \end{pmatrix}$

(i) Calculate **AB** [2]

(ii) Using the matrices **A** and **B**, demonstrate that the commutative law does not hold for matrix multiplication. [2]

(b) A system of equations is given by

$$\begin{aligned} \lambda x + y &= u \\ 3x - 2y + (\lambda - 3)z &= w \\ 10\lambda x + 3y - 2z &= 15 \end{aligned}$$

where u and w are real numbers.

(i) Find the values of λ for which the system has a unique solution. [5]

(ii) If $\lambda = 2$, find the necessary relationship between u and w if no real solution exists. [3]

(iii) If $\lambda = 2$, $u = 1$, $w = 4$, find the general solution to the system of equations. [4]

THIS IS THE END OF THE QUESTION PAPER