Rewarding Learning

ADVANCED

General Certificate of Education

2019

Mathematics

Assessment Unit F2

assessing

Module FP2: Further Pure Mathematics 2

AMF21

[AMF21]

MONDAY 17 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.
Answer **all eight** questions.
Show clearly the full development of your answers.
Answers should be given to three significant figures unless otherwise stated.
You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
A copy of the **Mathematical Formulae and Tables booklet** is provided.
Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all eight questions.

Show clearly the development of your answers.

Answers should be given to three significant figures unless otherwise stated.

1 Find, in terms of n, the sum of the series

$$1^2 - 2^2 + 3^2 - 4^2 + \ \dots\dots\dots \ + (2n-1)^2 - (2n)^2$$ [4]

2 Find, in radians, the general solution of the equation

$$6\tan^2\theta = 4\sin^2\theta + 1$$ [6]

3 The sequence $u_1, u_2, u_3, \dots\dots\dots$ is defined by

$$u_1 = 3 \qquad u_{n+1} = 4u_n + 1$$

Using mathematical induction, prove that

$$u_n = \frac{1}{3}[5(2^{2n-1}) - 1] \qquad \text{for all } n \geqslant 1$$ [5]

4 **(i)** Obtain the general solution of the differential equation

$$\tan x \frac{\mathrm{d}y}{\mathrm{d}x} + y = \sin x \tan x$$ [6]

(ii) Hence find the particular solution, given that $y = \dfrac{1}{2\sqrt{2}}$ when $x = \dfrac{\pi}{4}$ [2]

5 **(i)** Using partial fractions, show that

$$\frac{7x+4}{(1-3x^2)(2+3x)} \equiv \frac{2x+1}{(1-3x^2)} + \frac{2}{(2+3x)}$$ [5]

(ii) Hence find a series expansion for

$$\frac{7x+4}{(1-3x^2)(2+3x)}$$

up to and including the term in x^3 [5]

(iii) Find the range of values of x for which this expansion is valid. [3]

6 **(a)** One root of the equation

$$z^3 + az^2 + bz - 20 = 0$$

is $2 + \mathrm{i}$, where a and b are integers.

Find the other two roots and the values of a and b. [6]

(b) **(i)** Solve the equation

$$z^6 = 64$$

giving your answers in $r\mathrm{e}^{\mathrm{i}\theta}$ form. [5]

(ii) Sketch on an Argand diagram the hexagon whose vertices represent the solutions to

$$z^6 = 64$$ [2]

(iii) State the length of the sides of this regular hexagon. [1]

7

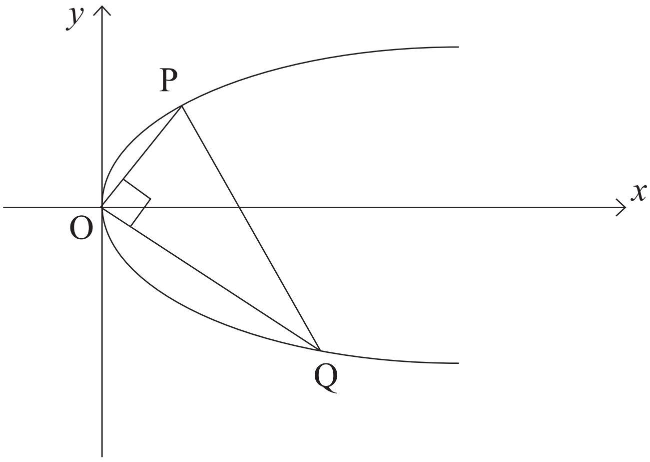

Fig. 1

Fig. 1 above shows the chord PQ of the parabola $y^2 = 4ax$
PQ subtends a right angle at the origin O.

Find the Cartesian equation of the locus of the midpoint of this chord. [8]

8 **(i)** I represents a measure of electric current after a time t.
I can be modelled by the differential equation

$$\frac{d^2 I}{dt^2} + 5\frac{dI}{dt} + 6I = 2\cos t - \sin t$$

Find the general solution of this equation. [9]

(ii) If when $t = 0$ $I = 0$ and $\frac{dI}{dt} = \frac{1}{2}$ show that

$$10I = 2e^{-3t} - 5e^{-2t} + 3\cos t + \sin t$$ [4]

(iii) Using **(ii)** deduce that, as time increases, the measure of current is approximated by the periodic function

$$\frac{1}{\sqrt{10}}\cos(t - \alpha) \qquad \text{where } \tan\alpha = \frac{1}{3}$$ [4]

THIS IS THE END OF THE QUESTION PAPER