



Rewarding Learning

ADVANCED

General Certificate of Education

2019

Mathematics

Assessment Unit M3

assessing

Module M3: Mechanics 3



AMM31

[AMM31]

THURSDAY 20 JUNE, MORNING

TIME

1 hour 30 minutes.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number on the Answer Booklet provided.

Answer **all six** questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

You are permitted to use a graphic or scientific calculator in this paper.

INFORMATION FOR CANDIDATES

The total mark for this paper is 75

Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.

Answers should include diagrams where appropriate and marks may be awarded for them.

Take $g = 9.8 \text{ m s}^{-2}$, unless specified otherwise.

A copy of the **Mathematical Formulae and Tables booklet** is provided.

Throughout the paper the logarithmic notation used is $\ln z$ where it is noted that $\ln z \equiv \log_e z$

Answer all six questions.

Show clearly the full development of your answers.

Answers should be given to three significant figures unless otherwise stated.

- 1 Part of a logo on an advertising hoarding can be modelled by the uniform lamina ABCDEF shown in **Fig. 1** below.

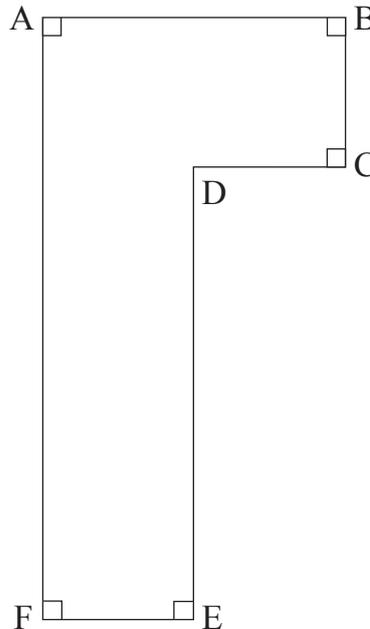


Fig. 1

The total mass of the lamina is 5 kg.
 $AF = 4\text{ m}$, $AB = 2\text{ m}$, $BC = 1\text{ m}$, $FE = 1\text{ m}$.

- (i) Find the distance of the centre of mass of the lamina from the edges AF and AB. [6]

Particles each of mass 1 kg are fastened at the points A, B, C and D.

- (ii) Find the distance of the centre of mass of this new system from the edges AF and AB. [4]

The new system is freely suspended from the point A and hangs in equilibrium.

- (iii) Find the angle the edge AF makes with the vertical. [3]

- 2 A light aircraft flies at 150 km h^{-1} in still air.
The wind is blowing steadily at 30 km h^{-1} from due west.
A is $200 \text{ km N}30^\circ\text{E}$ from B.
- (i) Find the course the pilot of the aircraft should steer to fly directly from A to B. [5]
- (ii) Find the time taken for this flight. [3]
- (iii) Find the course the pilot should steer to return directly from B to A. [3]
- 3 A particle P of mass $m \text{ kg}$ is suspended from a fixed point A by a light elastic string of natural length l metres and modulus of elasticity $2mg \text{ N}$.
P hangs in equilibrium a distance d metres vertically below A.
- (i) Show that $d = \frac{3l}{2}$ [4]
- Take the gravitational potential energy to be zero at the level of the horizontal through A.
- (ii) Find the total mechanical energy of the string and the particle, when the particle is hanging in equilibrium. [4]
- P is now pulled downwards until it is a distance $\frac{5l}{2}$ metres vertically below A and is released from rest.
- (iii) Find the speed of P when it is again a distance d vertically below A. [5]

- 4 (a) A particle P of mass 3 kg moves along a straight horizontal line under the action of a force F newtons;

$$F = \frac{1}{9} (30x - x^2)$$

where x metres is the distance of P from a fixed point on the line.

- (i) Find the work done by F as P moves from $x = 9$ to $x = 18$ [4]

When $x = 9$ the speed of P is 5 m s^{-1}

- (ii) Find the speed of P when $x = 18$ [3]

- (b) A particle Q of mass 0.6 kg moves under the action of forces A newtons and B newtons.

$$A = \begin{pmatrix} 2 \\ -1 \\ 4 \end{pmatrix} \quad \text{and} \quad B = \begin{pmatrix} \alpha + 3 \\ \alpha - 1 \\ \alpha + 2 \end{pmatrix}$$

Q moves from the origin O to the point L whose position vector is $\begin{pmatrix} 6 \\ 4 \\ -3 \end{pmatrix}$ metres.

- (i) Find, in terms of α , the total work done as Q moves from O to L. [4]

Q is initially at rest at O.

When Q reaches L it is moving at 12 m s^{-1}

- (ii) Find α . [3]

- 5 A particle moves with Simple Harmonic Motion between two fixed points A and B on a straight horizontal line.
A and B are a distance d metres apart.
The particle is instantaneously at rest at A and at B.
The maximum speed of the particle is 10 m s^{-1} and its maximum acceleration is 25 m s^{-2}

(i) Find d and the periodic time T of the motion. [6]

The particle passes through the point C on the line, where $BC = 2 \text{ m}$, while travelling towards A.

(ii) Find the speed and acceleration of the particle at C. [3]

(iii) Find the time taken for the particle to return to C for the first time. [4]

- 6 **Fig. 2** below shows a particle P of mass m kg at rest on a rough horizontal surface. P is attached to a fixed point A on the surface by a light elastic string of natural length l metres and modulus of elasticity $3mg$ newtons. A light inextensible string is fastened to P, passes over a fixed smooth pulley B at the edge of the surface and supports a second particle Q. Q is of mass m kg and hangs vertically below B.

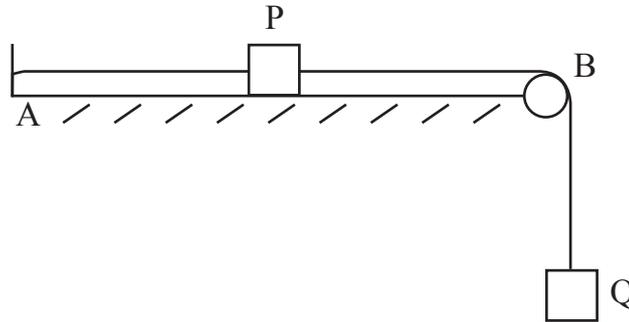


Fig. 2

When the distance AP is d metres, P is about to slide towards B.

When the distance AP is $\frac{3d}{2}$ metres, P is about to slide towards A.

(i) Find μ , the coefficient of friction between P and the surface. [9]

(ii) Find d in terms of l . [2]

THIS IS THE END OF THE QUESTION PAPER
