New Specification

Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2018

Further Mathematics

Unit 1 **(With calculator)**
Pure Mathematics

[GFM11]

GFM11

TUESDAY 12 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page.
Complete in black ink only. **Do not write with a gel pen.**
All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all twelve** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page 2.

Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log(ab) = \log a + \log b$

$\log\left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

1 If $y = \frac{1}{4}x^5 - \frac{6}{x^2} + 10$

(i) find $\frac{dy}{dx}$

Answer _____ [3]

(ii) **Hence** find $\frac{d^2y}{dx^2}$

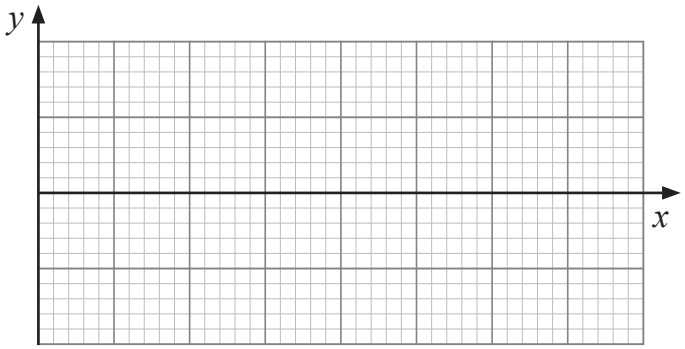
Answer _____ [2]

[Turn over

2 Find $\int_1^2 \left(3x^2 - \frac{2}{x^2} + 1\right) dx$

Answer _____ [5]

3 **(a)** Sketch the graph of $y = \cos x$ for $0° \leqslant x \leqslant 360°$ on the grid below.



[2]

(b) Solve the equation

$$\cos\left(\frac{4}{5}x + 30°\right) = -0.5$$

for $0° \leqslant x \leqslant 360°$

Answer _____ [5]

[Turn over

4 Solve the inequality $2x^2 + 3x - 4 \leqslant 5$

Answer _____ [5]

5 Matrices **P**, **Q** and **R** are defined by

$$\mathbf{P} = \begin{bmatrix} 3 & -1 \\ 2 & 8 \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} \text{ and } \mathbf{R} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

Find the matrix **X** such that

$$(\mathbf{P} - \mathbf{Q})\,\mathbf{X} = \mathbf{R}$$

Answer _____ [7]

[Turn over

6 Solve the equation

$$3^{2x-1} = 7^{x+2}$$

Answer _____ [5]

7 Find the **coordinates** of the point at which the tangent to the curve

$$y = \frac{5}{2}x^2 + 6 - \frac{40}{x}$$

is horizontal.

Answer _____ [6]

[Turn over

8 **(i)** Express

$$\frac{x^2 + 2x - 3}{x(x + 3)} + \frac{2x}{x + 1}$$

as a single fraction in its **simplest** form.

Answer _____ [5]

(ii) Hence show that the equation

$$\frac{x^2 + 2x - 3}{x(x + 3)} + \frac{2x}{x + 1} = 2$$

can be written as

$$x^2 - 2x - 1 = 0$$

[2]

(iii) Solve the equation $x^2 - 2x - 1 = 0$ using the method of **completing the square**.

Give your answer in the form $a \pm \sqrt{b}$, where a and b are whole numbers.

Answer _____ [3]

BLANK PAGE

DO NOT WRITE ON THIS PAGE

(Questions continue overleaf)

9 A curve is defined by the equation $y = x^3 - 6x^2 + 9x$

(i) Find the coordinates of the points where the curve meets the x-axis.

Answer _____ [3]

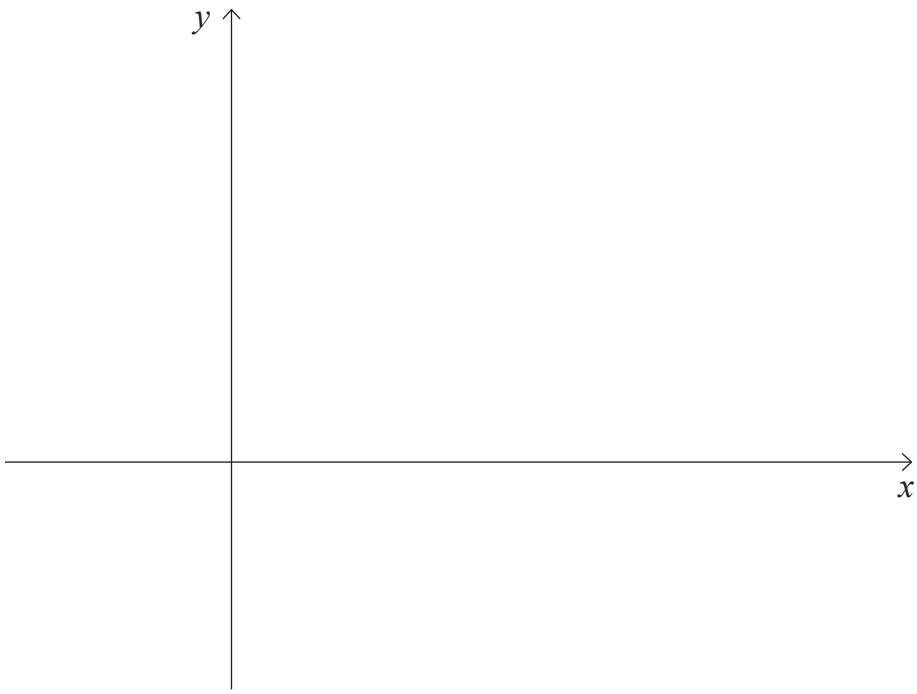
(ii) Find the coordinates of the turning points of the curve.

Answer _____ [5]

(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _____ [2]

(iv) Sketch the curve on the axes below.



[2]

[Turn over

10 A cycle shop sold 10 bikes over one weekend.

5 mountain bikes were sold at a cost of £x each.

3 children's bikes were sold at a cost of £y each.

2 road bikes were sold at a cost of £z each.

The total income from the weekend was £1985

(i) Write down an equation connecting x, y and z.

Answer _____ [1]

The cost of a road bike was £55 more than the sum of the cost of a mountain bike and a child's bike.

(ii) Write down a second equation connecting x, y and z.

Answer _____ [1]

The following weekend the cycle shop had a sale.

A mountain bike was reduced by £30 and a child's bike was reduced by £35. The cost of a road bike remained the same.

The shop sold 7 mountain bikes, 5 children's bikes and 1 road bike in the sale.

The total income from the sale was £1840

(iii) Show that

$$7x + 5y + z = 2225$$

[2]

[Turn over

(iv) Solve these equations to find the **original** cost of each type of bike.

Answer Mountain bike £ _____

Child's bike £ _____

Road bike £ _____ [8]

To make way for a newer model, the shop needed to sell off its remaining stock of 4 road bikes.

The total income from the sale of all 4 road bikes was £980

(v) By what **percentage** was the cost of a road bike reduced?

Answer _____ % [2]

[Turn over

11 David ran 5 marathons, each 26.2 miles long. He recorded the number of training runs, n, he completed and the time taken in minutes, T, for each marathon. The results are shown in the table below.

Marathon	**Training runs** n	**Time taken** T **(minutes)**		
1	48	315.44		
2	54	278.70		
3	62	241.10		
4	69	215.49		
5	76	194.70		

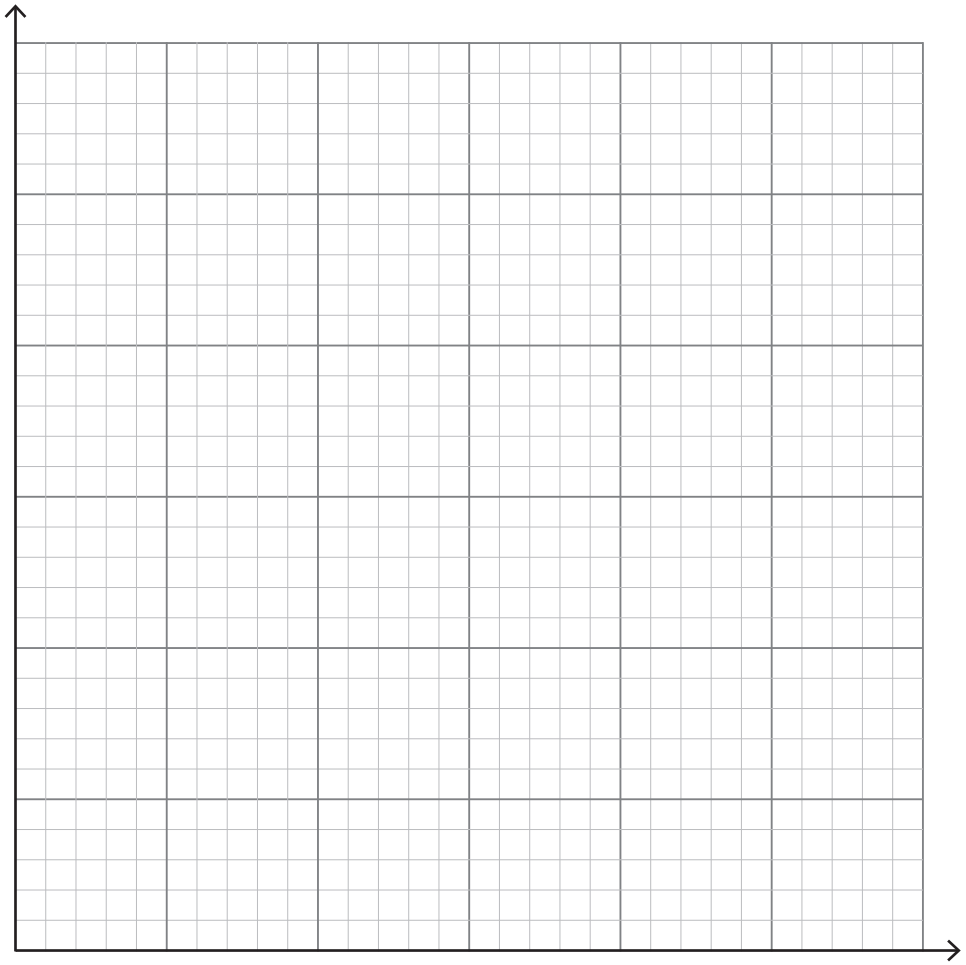
David believes that a relationship of the form

$$T = An^k$$

exists, where A and k are constants.

(i) Verify that a relationship of the form $T = An^k$ exists by drawing a suitable straight line graph on the grid below.

Show clearly the values used, correct to 3 decimal places, in the table opposite.



[7]

[Turn over

(ii) Hence find the values of A and k, correct to 2 decimal places.

Answer A = _____ , k = _____ [4]

David planned to complete 60 training runs before the Belfast marathon.

(iii) Use the formula $T = An^k$ with your values for A and k to calculate the time in which he could expect to complete the Belfast marathon.

Answer _____ minutes [1]

A runner must complete a marathon in less than 3 hours 5 minutes in order to qualify for the London marathon.

David wants his time in the Belfast marathon to qualify him for the London marathon.

(iv) Use the formula $T = An^k$ with your values for A and k to calculate the minimum number of training runs David should complete.

State any assumption which you make.

Answer _____ [2]

Assumption _____

_____ [1]

[Turn over

12 A supermarket chain wishes to build a store M somewhere on the straight road between two towns A and B.

If the store is built x km from A then, according to a survey, the number of people, N_A, from town A who will shop regularly each week at the store can be modelled by

$$N_A = 2000\left(1 - \frac{x^2}{100}\right)$$

Similarly, if the store is y km from B, then the number, N_B, from town B can be modelled by

$$N_B = 1000\left(1 - \frac{y^2}{200}\right)$$

(i) Show that the total number of regular weekly shoppers, N, from both towns is given by

$$N = 3000 - 20x^2 - 5y^2$$

[2]

The distance between A and B is 8 km.

(ii) Show that the total number of regular weekly shoppers from both towns is given by

$$N = 2680 + 80x - 25x^2$$

[3]

[Turn over

The supermarket chain wishes to position the store so that the total number of regular weekly shoppers is a maximum.

(iii) Find how far from town A the store should be built to maximise the total number of regular weekly shoppers, showing that this number is a maximum.

Answer _____ km [4]

THIS IS THE END OF THE QUESTION PAPER

BLANK PAGE

DO NOT WRITE ON THIS PAGE

DO NOT WRITE ON THIS PAGE

For Examiner's use only	
Question Number	**Marks**
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
Total Marks	

Examiner Number