

New
Specification



Rewarding Learning

**General Certificate of Secondary Education
2018**

Further Mathematics

**Unit 1 (With calculator)
Pure Mathematics**

[GFM11]

TUESDAY 12 JUNE, MORNING

**MARK
SCHEME**

General Marking Instructions

Introduction

Mark schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of students in schools and colleges.

The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes, therefore, are regarded as part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

$$1 \quad (i) \quad y = \frac{1}{4}x^5 - \frac{6}{x^2} + 10$$

$$= \frac{1}{4}x^5 - 6x^{-2} + 10$$

$$\frac{dy}{dx} = \frac{5}{4}x^4 + 12x^{-3} \quad \text{or} \quad \frac{5}{4}x^4 + \frac{12}{x^3}$$

3 × MW1

$$(ii) \quad \frac{d^2y}{dx^2} = 5x^3 - 36x^{-4} \quad \text{or} \quad 5x^3 - \frac{36}{x^4}$$

2 × MW1

5

$$2 \quad \int_1^2 \left(3x^2 - \frac{2}{x^2} + 1 \right) dx$$

$$= \int_1^2 (3x^2 - 2x^{-2} + 1) dx$$

$$= \left[x^3 + \frac{2}{x} + x \right]_1^2$$

3 × MW1

$$= [8 + 1 + 2] - [1 + 2 + 1]$$

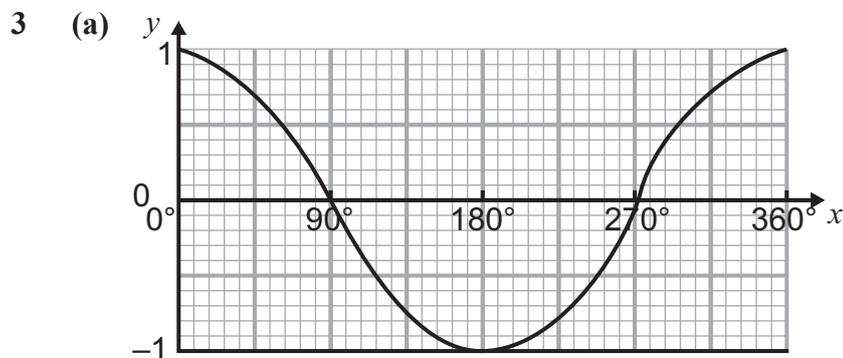
M1

$$= 11 - 4$$

$$= 7$$

W1

5



M1 (accuracy)

W1 (curve)

$$(b) \quad \cos \left(\frac{4}{5}x + 30^\circ \right) = -0.5$$

$$\frac{4}{5}x + 30^\circ = 120^\circ$$

or

$$240^\circ$$

MW1 MW1

$$\frac{4}{5}x + 30^\circ = 120^\circ$$

or

$$\frac{4}{5}x + 30^\circ = 240^\circ$$

M1

$$\frac{4}{5}x = 90^\circ$$

or

$$\frac{4}{5}x = 210^\circ$$

$$x = 112.5^\circ$$

or

$$x = 262.5^\circ$$

W1 W1

7

4 $2x^2 + 3x - 4 \leq 5$

$2x^2 + 3x - 9 \leq 0$

Find roots

$2x^2 + 3x - 9 = 0$

$(2x - 3)(x + 3) = 0$

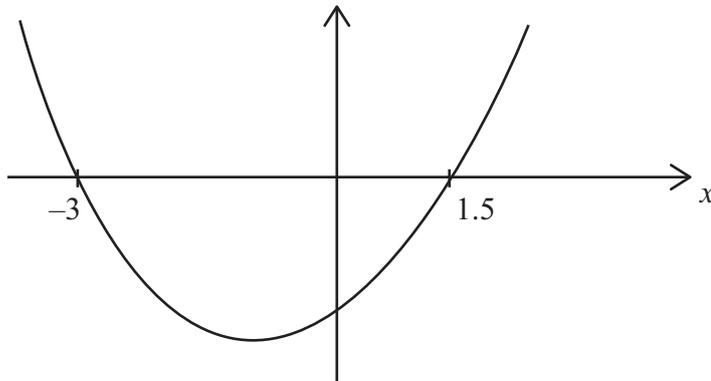
$x = 1.5$ or $x = -3$

MW1

M1

W1

Sketch:



Alternatively:

	-3	1.5	
$2x - 3$	-	-	+
$x + 3$	-	+	+
$(2x - 3)(x + 3)$	+	-	+

$-3 \leq x \leq 1.5$

W1 W1

5

5 Let $\mathbf{A} = \mathbf{P} - \mathbf{Q}$

$$\mathbf{A} = \begin{bmatrix} 3 & -1 \\ 2 & 8 \end{bmatrix} - \begin{bmatrix} 1 & -2 \\ -3 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 1 \\ 5 & 4 \end{bmatrix}$$

M1 W1

$\mathbf{AX} = \mathbf{R}$

$\mathbf{X} = \mathbf{A}^{-1} \mathbf{R}$

$$\mathbf{A}^{-1} = \frac{1}{3} \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix}$$

M1 MW1 MW1

$$\mathbf{X} = \frac{1}{3} \begin{bmatrix} 4 & -1 \\ -5 & 2 \end{bmatrix} \begin{bmatrix} 4 \\ 1 \end{bmatrix}$$

M1

$$= \frac{1}{3} \begin{bmatrix} 15 \\ -18 \end{bmatrix} = \begin{bmatrix} 5 \\ -6 \end{bmatrix}$$

W1

7

6 $3^{2x-1} = 7^{x+2}$

$$\log 3^{2x-1} = \log 7^{x+2}$$

M1

$$(2x - 1) \log 3 = (x + 2) \log 7$$

MW1

$$2x \log 3 - \log 3 = x \log 7 + 2 \log 7$$

W1

$$2x \log 3 - x \log 7 = 2 \log 7 + \log 3$$

$$x(2 \log 3 - \log 7) = 2 \log 7 + \log 3$$

M1

$$x = \frac{2 \log 7 + \log 3}{2 \log 3 - \log 7}$$

$$x = 19.857 \rightarrow 19.86$$

W1

Alternative Solution

$$2x - 1 = \log_3 7^{x+2}$$

M1

$$2x - 1 = (x + 2) \log_3 7$$

MW1

$$2x - x \log_3 7 = 1 + 2 \log_3 7$$

W1

$$x(2 - \log_3 7) = 1 + 2 \log_3 7$$

$$x = \frac{1 + 2 \log_3 7}{2 - \log_3 7}$$

M1

$$= \frac{1 + 2(1.771243749)}{2 - 1.771243749}$$

$$= 19.86$$

W1

(or similarly if $\log_7 3^{2x-1} = x + 2$)

AVAILABLE
MARKS

5

9 (i) $x^3 - 6x^2 + 9x = 0$

$$x(x^2 - 6x + 9) = 0$$

M1

$$x(x - 3)^2 = 0$$

MW1

$$x = 0 \text{ or } 3$$

Points are (0, 0), (3, 0)

W1

(ii) $\frac{dy}{dx} = 3x^2 - 12x + 9$

M1 W1

$$= 0$$

M1

$$x^2 - 4x + 3 = 0$$

$$(x - 1)(x - 3) = 0$$

$$x = 1 \text{ or } 3$$

W1

$$y = 4 \text{ or } 0$$

Turning points at (1, 4) and (3, 0)

W1

(iii) $\frac{d^2y}{dx^2} = 6x - 12$

MW1

When $x = 1$, $\frac{d^2y}{dx^2} = -6 < 0 \therefore \text{max}$

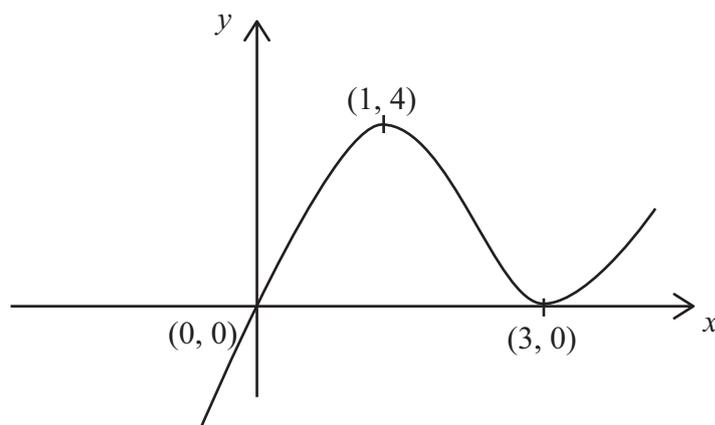
When $x = 3$, $\frac{d^2y}{dx^2} = 6 > 0 \therefore \text{min}$

Minimum at (3, 0)

Maximum at (1, 4)

MW1

(iv)



M1 (shape)
W1 (points)

12

- 10 (i) $5x + 3y + 2z = 1985$ W1
- (ii) $z = x + y + 55$
 $-x - y + z = 55$ W1
- (iii) $7(x - 30) + 5(y - 35) + z = 1840$ M1
 $7x - 210 + 5y - 175 + z = 1840$ W1
 $7x + 5y + z = 2225$
- (iv) $5x + 3y + 2z = 1985$ (1)
 $-x - y + z = 55$ (2)
 $7x + 5y + z = 2225$ (3)
- $5x + 3y + 2z = 1985$ (1)
 $-2x - 2y + 2z = 110$ (2) $\times 2$
 $\hline 7x + 5y = 1875$ (4) M1 W1
- $7x + 5y + z = 2225$ (3)
 $-x - y + z = 55$ (2)
 $\hline 8x + 6y = 2170$ (5) M1 W1
- $42x + 30y = 11\,250$ (4) $\times 6$
 $40x + 30y = 10\,850$ (5) $\times 5$
 $\hline 2x = 400$
 $x = 200$ M1 W1
- Back substitution $y = 95, z = 350$ M1
Mountain bike £200
Child's bike £95
Road bike £350 W1
- (v) $\pounds 980 \div 4 = \pounds 245$
 $\pounds 350 - \pounds 245 = \pounds 105$ reduction
 $\frac{105}{350} \times 100 = 30\%$ M1 W1

Alternative solution

Original cost of 4 bikes = $350 \times 4 = \pounds 1400$
Reduction = $1400 - 980 = 420$
Percentage reduction = $\frac{420}{1400} \times 100 = 30\%$

M1 W1

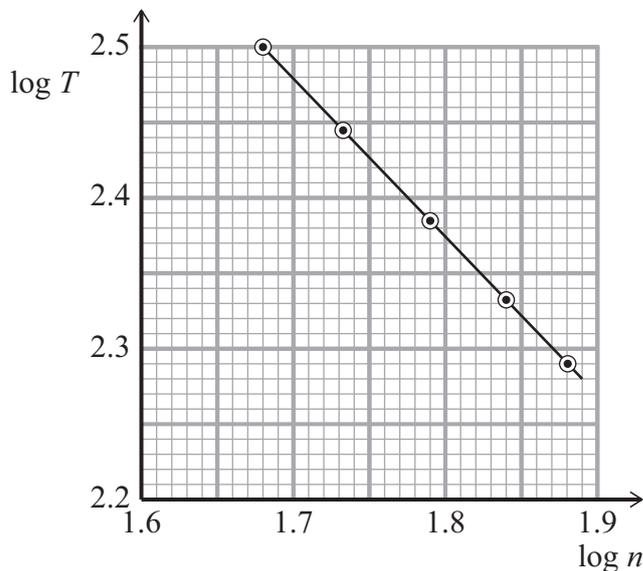
14

- 11 (i) $T = An^k$
 $\log T = \log An^k = \log A + \log n^k$
 $\log T = k \log n + \log A$

M1

Marathon	Training runs, n	Time taken, T	$\log n$	$\log T$
1	48	315.44	1.681	2.499
2	54	278.70	1.732	2.445
3	62	241.10	1.792	2.382
4	69	215.49	1.839	2.333
5	76	194.70	1.881	2.289

M1 (taking logs)

W1 (all $\log n$ answers correct to 3 dp)W1 (all $\log T$ answers correct to 3 dp)

W1 (labels)

W1 (points plotted correctly)

W1 (straight line through points)

(ii) $k = \frac{2.289 - 2.499}{1.881 - 1.681} = \frac{-0.21}{0.2} = -1.05$

M1 W1

$$An^k = T$$

$$A(48)^{-1.05} = 315.44$$

M1

$$0.017167A = 315.44$$

$$A = 18374.692 \rightarrow 18374.69$$

W1

- (iii) $T = An^k$
 $T = 18374.69 \times 60^{-1.05}$
 $T = 249.55$ minutes

MW1

AVAILABLE
MARKS

(iv) $18374.69 n^{-1.05} = 185$

$$n^{-1.05} = 0.010068197$$

$$n = (0.010068197)^{\frac{-1}{1.05}}$$

(or $-1.05 \log n = \log 0.010068197$)

$$n = 79.79 \rightarrow 80 \text{ training runs}$$

Assumption: The relationship holds outside the range

12 (i) $N = N_A + N_B$

$$= 2000 \left(1 - \frac{x^2}{100}\right) + 1000 \left(1 - \frac{y^2}{200}\right)$$

$$= 2000 - 20x^2 + 1000 - 5y^2$$

$$= 3000 - 20x^2 - 5y^2$$

(ii) $x + y = 8$

$$y = 8 - x$$

$$N = 3000 - 20x^2 - 5(8 - x)^2$$

$$= 3000 - 20x^2 - 5(64 - 16x + x^2)$$

$$= 3000 - 20x^2 - 320 + 80x - 5x^2$$

$$= 2680 + 80x - 25x^2$$

(iii) $\frac{dN}{dx} = 80 - 50x$

$$= 0 \text{ for max}$$

$$x = \frac{80}{50} = 1.6$$

$$\frac{d^2N}{dx^2} = -50 < 0 \therefore \text{max}$$

Store should be built 1.6 km from A

AVAILABLE
MARKS

M1

W1

M1

15

M1

W1

MW1

M1

W1

MW1

M1

W1

MW1

9

Total

100