



**General Certificate of Secondary Education**  
**2019**

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## **Further Mathematics**

**Unit 1**  
**Pure Mathematics**

**[GFM11]**

**FRIDAY 14 JUNE, AFTERNOON**

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**MARK  
SCHEME**

## General Marking Instructions

### Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

**M** indicates marks for correct method.

**W** indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

**MW** indicates marks for combined method and accurate working.

### Assessment Objectives

Below are the assessment objectives for GCSE Further Mathematics.

#### Use and apply standard techniques (AO1)

Candidates should be able to:

- accurately recall facts, terminology and definitions;
- use and interpret notation correctly; and
- accurately carry out routine procedures or set tasks requiring multi-step solutions.

#### Reason, interpret and communicate mathematically (AO2)

Candidates should be able to:

- make deductions, inferences and draw conclusions from mathematical information;
- construct chains of reasoning to achieve a given result;
- present arguments and proofs; and
- assess the validity of an argument and critically evaluate a given way of presenting information.

#### Solve problems within mathematics and in other contexts

Candidates should be able to:

- translate problems in mathematical or non-mathematical contexts into a process or a series of mathematical processes;
- make and use connections between different parts of mathematics;
- interpret results in the context of the given problem;
- evaluate methods used and results obtained; and
- evaluate solutions to identify how they may have been affected by assumptions made.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

### **Positive marking:**

It is our intention to reward candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

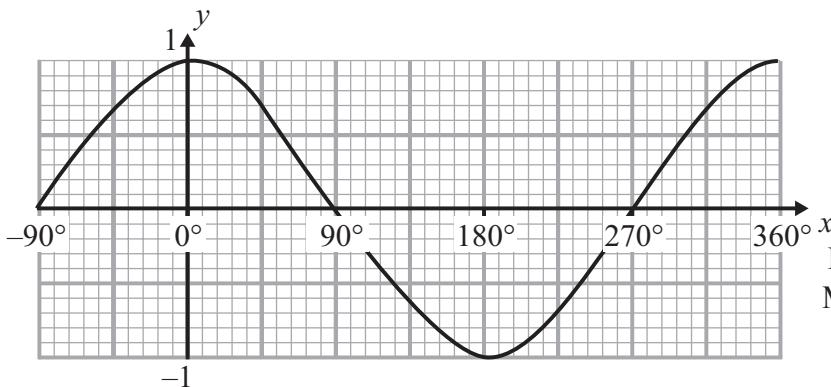
Some common examples of this occur in the following cases:

- (a)** a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b)** readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

		AVAILABLE MARKS
1	$A^2 = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 18 \\ -6 & 13 \end{bmatrix}$	M1 W1
	$B - A^2 = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix} - \begin{bmatrix} 1 & 18 \\ -6 & 13 \end{bmatrix}$	
	$= \begin{bmatrix} 0 & -20 \\ 3 & -8 \end{bmatrix}$	M1 W1
2	(i) $f(x) = x^2 - x + 4$ $= (x - \frac{1}{2})^2 - \frac{1}{4} + 4$ $= (x - \frac{1}{2})^2 + \frac{15}{4}$	M1 W1
	(ii) Minimum value = $\frac{15}{4}$	W1
	Occurs when $x = \frac{1}{2}$	W1
3	(a) $y = \frac{3x^2}{8} + \frac{5}{x^4} - 12x$ $= \frac{3}{8}x^2 + 5x^{-4} - 12x$ $\frac{dy}{dx} = \frac{3}{4}x - 20x^{-5} - 12$ or $\frac{3}{4}x - \frac{20}{x^5} - 12$	$3 \times \text{MW1}$
	(b) $\int \left( \frac{3}{4x^2} - 2x^3 \right) dx$ $= \int \left( \frac{3}{4}x^{-2} - 2x^3 \right) dx$ $= -\frac{3}{4}x^{-1} - \frac{1}{2}x^4 + c$ or $-\frac{3}{4x} - \frac{1}{2}x^4 + c$	$3 \times \text{MW1}$

4 (a)



AVAILABLE MARKS

MW1 (shape)  
MW1 (points)

(b) (i)  $\cos x = -0.184$

$x = 100.603 \text{ or } 259.397$

$\rightarrow 100.6^\circ \text{ or } 259.4^\circ \text{ (to 1 d.p.)}$

MW1 MW1

(ii)  $\cos(2\theta - 15) = -0.184$

$2\theta - 15 = 100.603 \text{ or } 259.397$

$\theta = 57.8^\circ \text{ (outside range)}$

or  $137.2^\circ$

$\theta = 137.2^\circ \text{ (to 1 d.p.)}$

M1

W1

6

5  $\mathbf{P}\mathbf{X} = \mathbf{Q}$ 

$\mathbf{X} = \mathbf{P}^{-1} \mathbf{Q}$

$\mathbf{P}^{-1} = \frac{1}{10} \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix}$

MW1 MW1

$\mathbf{X} = \frac{1}{10} \begin{bmatrix} -4 & -2 \\ -1 & -3 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

M1

$= \frac{1}{10} \begin{bmatrix} -30 \\ -10 \end{bmatrix} = \begin{bmatrix} -3 \\ -1 \end{bmatrix}$

W1

4

6  $x^2 - 4x - 5 < 0$

Find the roots

$x^2 - 4x - 5 = 0$

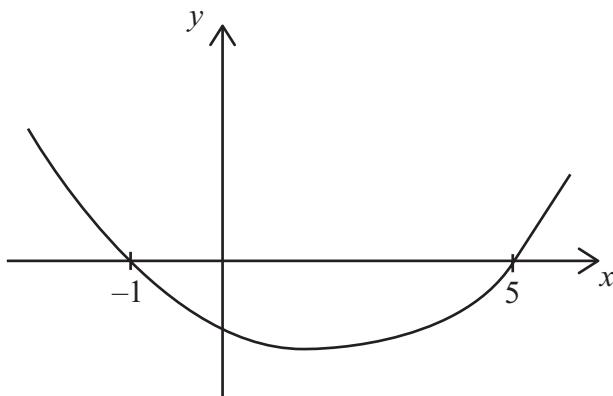
$(x + 1)(x - 5) = 0$

$x = -1 \text{ or } x = 5$

M1

MW1

W1



$-1 < x < 5$

W1

**Alternative solution**

$x^2 - 4x - 5 = 0$

$(x + 1)(x - 5) = 0$

$x = -1 \text{ or } x = 5$

M1

MW1

W1

	$x < -1$	$-1 < x < 5$	$x > 5$
$(x + 1)$	-	+	+
$(x - 5)$	-	-	+
$(x + 1)(x - 5)$	+	-	+

$-1 < x < 5$

W1

4

7  $2x + 3y + z = 5 \quad (1)$

$3x - 4y + 2z = -9 \quad (2)$

$x + 5y - 3z = 6 \quad (3)$

$(1) \times 2 - (2) \rightarrow x + 10y = 19 \quad (4)$

$(1) \times 3 + (3) \rightarrow 7x + 14y = 21$

i.e.  $x + 2y = 3 \quad (5)$

$(4) - (5) \rightarrow 8y = 16$

$y = 2$

From (5):  $x = 3 - 2y$

$= 3 - 4$

$= -1$

From (1):  $z = 5 - 2x - 3y$

$= 5 + 2 - 6$

$= 1$

M1 W1

M1 W1

M1

W1

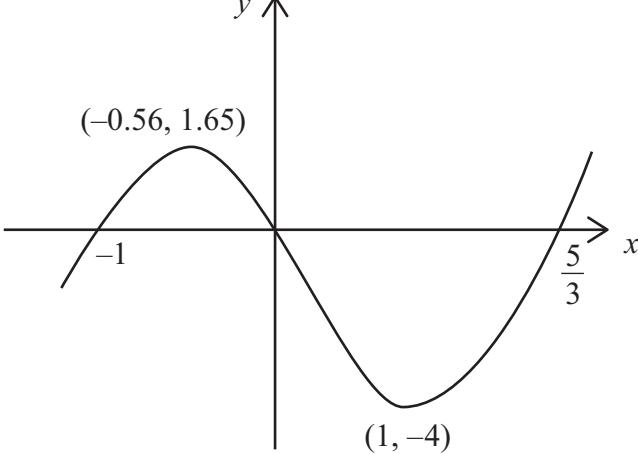
M1

W1

M1

W1

8

8	(i)	Meets $x$ -axis at $(0, 0)$ , $(\frac{5}{3}, 0)$ , $(-1, 0)$	3 × W1	AVAILABLE MARKS
	(ii)	$y = 3x^3 - 2x^2 - 5x$	W1	
		$\frac{dy}{dx} = 9x^2 - 4x - 5 = 0$	MW1 M1	
		$(9x + 5)(x - 1) = 0$	MW1	
		$x = -\frac{5}{9}$ or $x = 1$	W1	
		$y = \frac{400}{243}$ (or 1.65) or $y = -4$	W1	
		Turning points at $(-0.56, 1.65)$ , $(1, -4)$ or $(-\frac{5}{9}, \frac{400}{243})$ , $(1, -4)$		
	(iii)	$\frac{d^2y}{dx^2} = 18x - 4$	MW1	
		When $x = -\frac{5}{9}$ , $\frac{d^2y}{dx^2} = -14 < 0 \therefore$ maximum		
		When $x = 1$ , $\frac{d^2y}{dx^2} = 14 > 0 \therefore$ minimum	M1	
		Maximum at $(-0.56, 1.65)$		
		Minimum at $(1, -4)$		
	(iv)		M1 (shape) W1 (points)	13

		AVAILABLE MARKS
9	(a) $2 \log y = 3 \log x$ $\log y^2 = \log x^3$ $y^2 = x^3$ $y = x^{3/2}$ (or $\sqrt{x^3}$ )	M1  W1
	<b>Alternative solution</b> $\log y = \frac{3}{2} \log x$ $\log y = \log x^{3/2}$ $y = x^{3/2}$ (or $\sqrt{x^3}$ )	M1  W1
(b) (i)	$4 \times 2^x = 2^y$ $2^2 \times 2^x = 2^y$ $2^{x+2} = 2^y$ $y = x + 2$	MW1
(ii)	$6^{3x-2} = 4 \times 2^x$ $6^{3x-2} = 2^{x+2}$ (from (i)) $(3x-2) \log 6 = (x+2) \log 2$ (or $\log 4 + x \log 2$ ) $3x \log 6 - x \log 2 = 2 \log 2 + 2 \log 6$ $x(3 \log 6 - \log 2) = 2 \log 2 + 2 \log 6$ $x = \frac{2 \log 2 + 2 \log 6}{3 \log 6 - \log 2}$ $x = 1.06$	MW1, MW1  MW1  W1
		7

10 (i)  $V = ar^b$ 

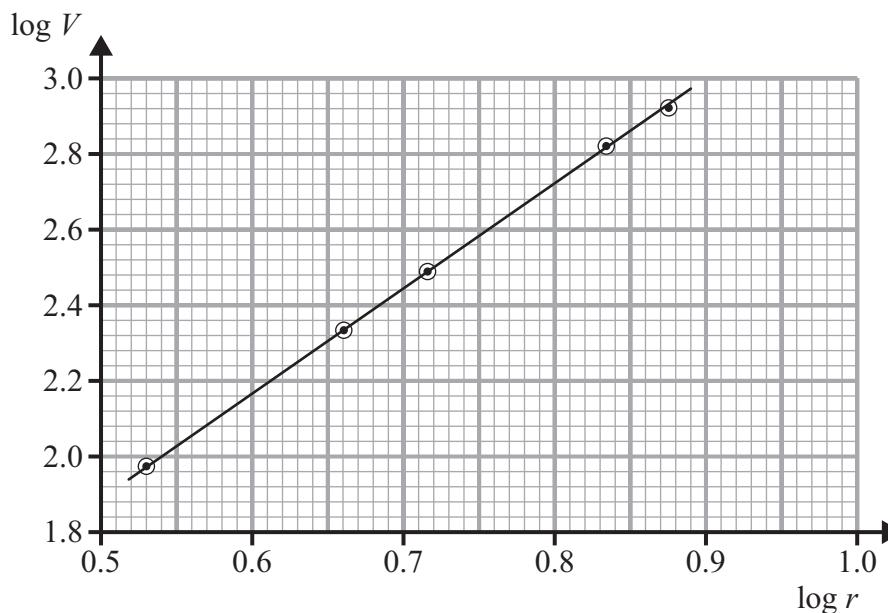
$$\log V = \log ar^b = b \log r + \log a$$

MW1

AVAILABLE MARKS

$\log r$	$\log V$
0.531	1.991
0.663	2.346
0.716	2.490
0.833	2.804
0.875	2.919

M1 (logs)  
W1 ( $\log r$  values)  
W1 ( $\log V$  values)



W1 (labels)  
W1 (points correct)  
W1 (straight line)

$$\text{Gradient} = b = \frac{2.919 - 1.991}{0.875 - 0.531} = 2.698 \rightarrow 2.7 \text{ (to 1 d.p.)}$$

M1 W1

$$V = ar^b$$

$$98.0 = a(3.4)^{2.698}$$

$$a = 3.6 \text{ (to 1 d.p.)}$$

M1 W1

Hence

$$V = 3.6r^{2.7}$$

$$\text{(ii)} \quad V = 3.6r^{2.7}$$

$$V = 3.6(5.6)^{2.7} = 377 \text{ cm}^3$$

MW1

$$\text{(iii)} \quad 3.6r^{2.7} = 1000$$

$$r = \left( \frac{1000}{3.6} \right)^{\frac{1}{2.7}}$$

$$= 8.037$$

MW1

$$\text{Diameter} = 2r = 16.074 \rightarrow 16 \text{ cm}$$

W1

Assumption: The relationship holds outside the range

M1

15

		AVAILABLE MARKS
11	$y = 2 - \frac{3}{x}$ When $y = 0, x = \frac{3}{2}$ Curve cuts $x$ -axis at $(\frac{3}{2}, 0)$ $\frac{dy}{dx} = 3x^{-2} = \frac{3}{x^2}$ When $x = \frac{3}{2}, \frac{dy}{dx} = \frac{4}{3}$ = gradient of tangent Gradient of normal = $-\frac{3}{4}$ Equation of normal is $y = -\frac{3}{4}x + c$ When $x = \frac{3}{2}, y = 0$ $\therefore 0 = -\frac{9}{8} + c$ $\therefore c = \frac{9}{8}$ Equation of normal is $y = -\frac{3}{4}x + \frac{9}{8}$	W1 MW1 W1 M1 MW1 MW1
12	(i) $(x - 4)(2x + 5) = 2x^2 - 3x - 20$ $(x + 3)(x - 4)(2x + 5) = (x + 3)(2x^2 - 3x - 20)$ $= 2x^3 - 3x^2 - 20x + 6x^2 - 9x - 60$ $= 2x^3 + 3x^2 - 29x - 60$  (ii) Numerator = $(2x^3 + 3x^2 - 29x - 60) - 2x^3 - 4x - 18$ $= 3x^2 - 33x - 78$ $\frac{3x^2 - 33x - 78}{x^2 - 13x} = \frac{3(x^2 - 11x - 26)}{x(x - 13)}$ $= \frac{3(x - 13)(x + 2)}{x(x - 13)}$ $= \frac{3(x + 2)}{x}$ (or $\frac{3x + 6}{x}$ )	MW1, MW1, MW1 MW1 MW1 MW1 W1
13	(i) Area A = $\int_0^1 (k + 2x - 3x^2)dx$ $= [kx + x^2 - x^3]_0^1$ $= [k + 1 - 1] - [0] = k$  Area B = $\int_0^2 (k + 2x - 3x^2)dx$ $= [kx + x^2 - x^3]_0^2$ $= [2k + 4 - 8] - [0] = 2k - 4$	M1 MW1 W1 M1 W1
	(ii) Area A = $\frac{5}{9}$ area B $k = \frac{5}{9} (2k - 4)$ $9k = 10k - 20$ $k = 20$	M1 W1

		AVAILABLE MARKS
14	(i) Area A = $xy + \frac{10}{9}xy$	MW1
(ii) Perimeter P = $2x + 2y + \frac{2y}{3}$ = $2x + \frac{8}{3}y$		MW1
(iii) P = $2x + \frac{8}{3}y = 96$ $3x + 4y = 144$ $y = 36 - \frac{3}{4}x$		M1 W1
(iv) A = $\frac{10}{9}x(36 - \frac{3}{4}x)$ = $40x - \frac{5}{6}x^2$ $\frac{dA}{dx} = 40 - \frac{10}{6}x = 0$ for maximum $x = 24$ $y = 18$ $\frac{d^2A}{dx^2} = -\frac{10}{6} < 0 \therefore$ maximum Adults' pool: 24m by 18m Children's pool: 8m by 6m		M1 W1 MW1 W1 MW1 W1 MW1 W1 W1
		10
	<b>Total</b>	<b>100</b>