Rewarding Learning

Centre Number

Candidate Number

General Certificate of Secondary Education
2019

Further Mathematics

Unit 1 **(With calculator)**
Pure Mathematics

[GFM11]

GFM11

FRIDAY 14 JUNE, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page.
Complete in black ink only. **Do not write with a gel pen.**
All working **must** be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all fourteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on page 2.

11900

Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Differentiation: If $y = ax^n$ then $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \,\mathrm{d}x = \frac{ax^{n+1}}{n+1} + c$ $(n \neq -1)$

Logarithms: If $a^x = n$ then $x = \log_a n$

$\log(ab) = \log a + \log b$

$\log\left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\det \mathbf{A} = ad - bc$

and $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $(ad - bc \neq 0)$

1 Matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 2 & 3 \\ -1 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 1 & -2 \\ -3 & 5 \end{bmatrix}$$

Find the value of $\mathbf{B} - \mathbf{A}^2$

Answer _____ [4]

[Turn over

2 A function $f(x)$ is defined by

$$f(x) = x^2 - x + 4$$

(i) Use the method of **completing the square** to rewrite $f(x)$ in the form

$$(x + a)^2 + b$$

where a and b are constants.

Answer _____ [2]

(ii) **Hence** find the minimum value of $f(x)$ and the value of x for which it occurs.

Answer Minimum value _____ [1]

when x = _____ [1]

3 **(a)** Find $\frac{dy}{dx}$ if $y = \frac{3}{8}x^2 + \frac{5}{x^4} - 12x$

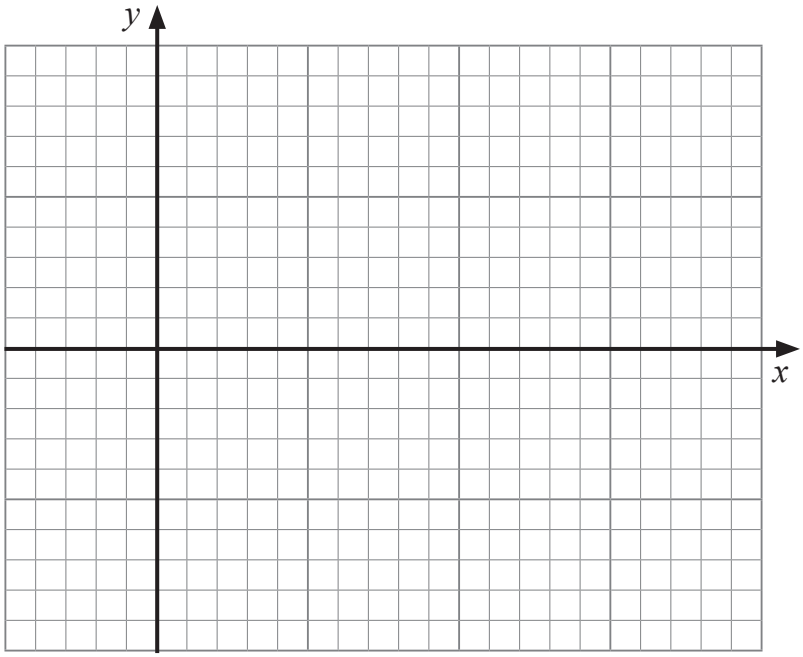
Answer _____ [3]

(b) Find $\int \left(\frac{3}{4x^2} - 2x^3\right) dx$

Answer _____ [3]

[Turn over

4 **(a)** Sketch the graph of $y = \cos x$ for $-90° \leqslant x \leqslant 360°$



[2]

(b) **(i)** Solve the equation

$$\cos x = -0.184$$

for $0° \leqslant x \leqslant 360°$

Give your answers correct to 1 decimal place.

Answers _____ [2]

(ii) Hence solve the equation

$$\cos(2\theta - 15°) = -0.184$$

for $90° \leqslant \theta \leqslant 180°$

Give your answer correct to 1 decimal place.

Answer _____ [2]

5 Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix} \text{ and } \mathbf{Q} = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$$

Using a matrix method, find the matrix **X** such that

$$\mathbf{PX} = \mathbf{Q}$$

Answer _____ [4]

6 Solve the inequality

$$x^2 - 4x - 5 < 0$$

You **must** show clearly each stage of your solution.

Answer _____ [4]

[Turn over

7 Solve the following set of simultaneous equations

$$2x + 3y + z = 5$$

$$3x - 4y + 2z = -9$$

$$x + 5y - 3z = 6$$

You **must** show clearly each stage of your solution.

Answer $x =$ _____ , $y =$ _____ , $z =$ _____ [8]

You may use this page for Question 7 if needed.

(Questions continue overleaf)

8 A curve is defined by the equation $y = x(3x - 5)(x + 1)$

(i) Write down the **coordinates** of the points where the curve meets the x-axis.

Answer _____ [3]

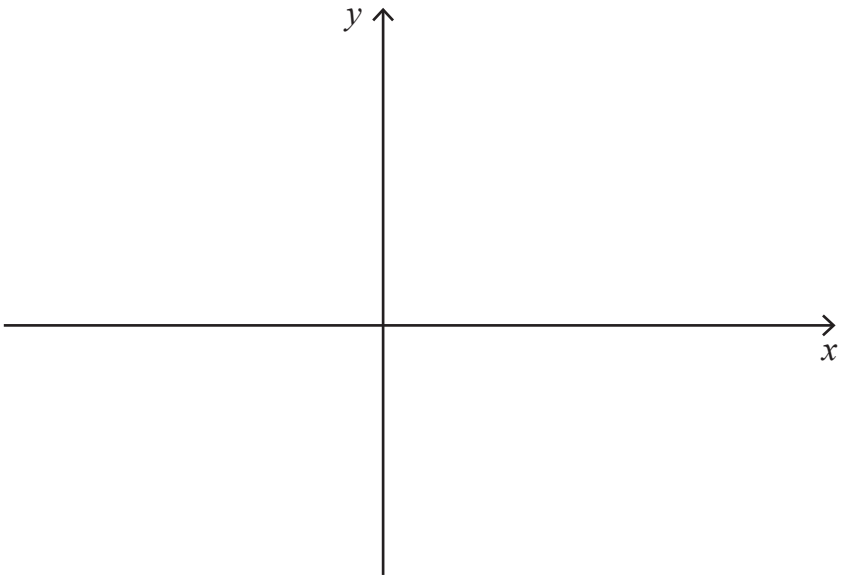
(ii) Find the coordinates of the turning points of the curve.

Answer _____ [6]

(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _____ [2]

(iv) Sketch the curve on the axes below.



[2]

[Turn over

9 **(a)** If $2 \log y = 3 \log x$ write y in terms of x.

Answer _____ [2]

(b) **(i)** If $4 \times 2^x = 2^y$ show that

$$y = x + 2$$

[1]

(ii) Hence or otherwise solve the equation

$$6^{3x-2} = 4 \times 2^{x}$$

Answer _____ [4]

[Turn over

10 Rory has a set of objects, each with a circular base. He records the base radius, r cm, and the volume, V cm^3, of 5 of these objects.

The results are given in the table below.

Base radius r (cm)	**Volume** V (cm^3)		
3.4	98.0		
4.6	221.7		
5.2	308.7		
6.8	636.9		
7.5	829.8		

Rory believes that a relationship of the form

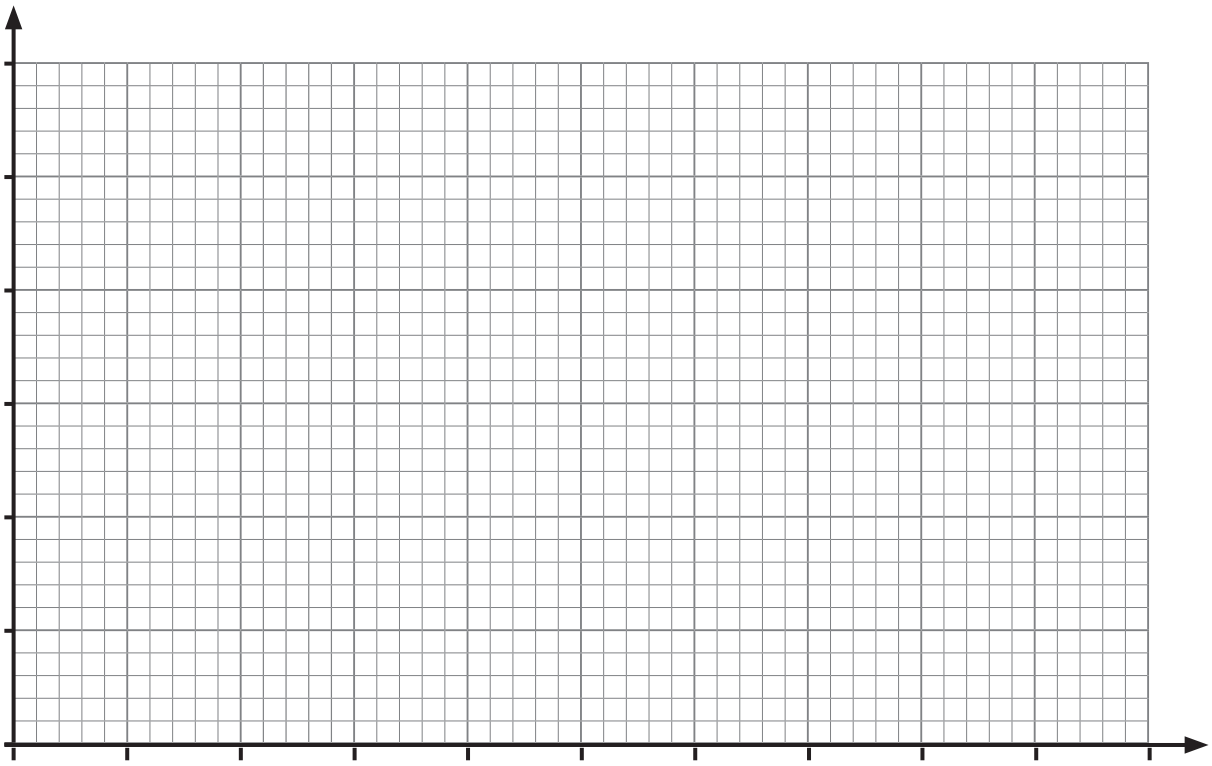
$$V = ar^b$$

exists, where a and b are constants.

(i) Verify that a relationship of the form $V = ar^b$ exists by drawing a suitable straight line graph on the grid opposite.

Show clearly the values used, correct to 3 decimal places, in the table above.

Hence find the values of a and b, correct to 1 decimal place.



Answer a = _____ , b = _____ [11]

[Turn over

Use the formula $V = ar^b$ with your values for a and b to calculate

(ii) the volume of an object with radius 5.6 cm, giving your answer to the nearest cm^3,

Answer _____ cm^3 [1]

(iii) the **diameter** of an object with a volume of 1000 cm^3.
Give your answer correct to the nearest cm and state any assumption that you make.

Answer _____ cm [2]

Assumption _____

_____ [1]

11 Find the equation of the **normal** to the curve

$$y = 2 - \frac{3}{x}$$

at the point where the curve cuts the ***x*-axis**.

Answer _____ [5]

[Turn over

12 (i) Expand and simplify the expression

$$(x + 3)(x - 4)(2x + 5)$$

Answer _____ [3]

(ii) Hence simplify fully the expression

$$\frac{(x + 3)(x - 4)(2x + 5) - 2x\,(x^2 + 2) - 18}{x^2 - 13x}$$

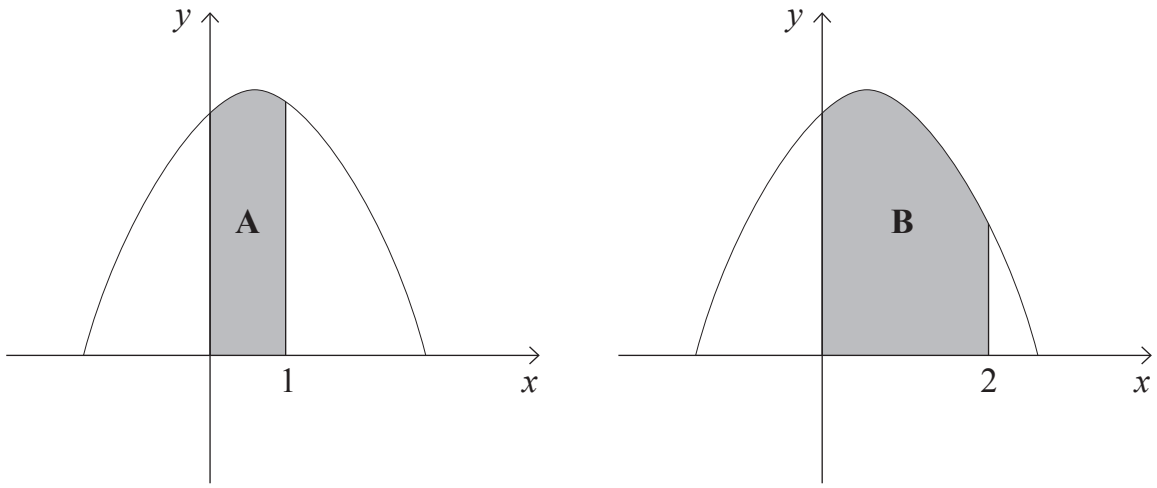
Answer _____ [4]

[Turn over

13 The diagrams below show sketches of the curve

$$y = k + 2x - 3x^2$$

where k is a constant.



(i) Find expressions in terms of k for the area of **A** and the area of **B**.

Answer Area of **A** _____ [3]

Area of **B** _____ [2]

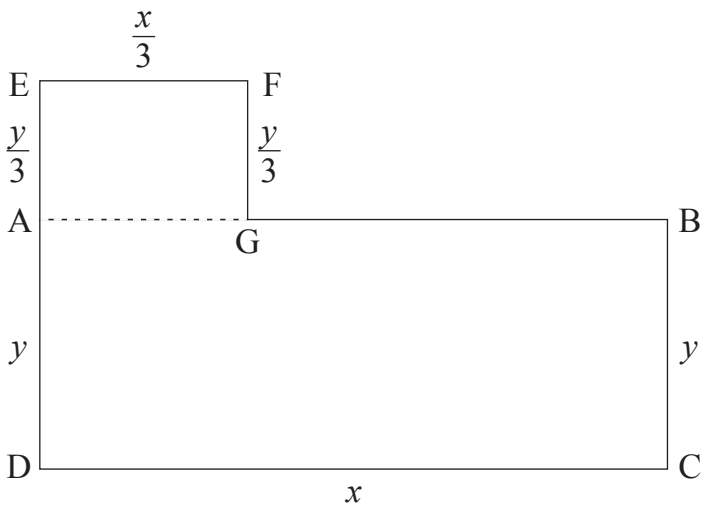
(ii) Given that the area of **A** is $\frac{5}{9}$ of the area of **B**, find the value of k.

Answer _____ [2]

14 The owners of a hotel wish to build a swimming pool in the hotel grounds. They plan to build a rectangular pool ABCD for adults, of length x m and width y m.

At one end of the pool they plan to add a children's pool AEFG, as shown in the diagram below.

The length and width of the children's pool are to be $\frac{1}{3}$ of the length and width of the adults' pool, respectively.



Write down, in terms of x and y,

(i) the total area of the two pools,

Answer _____ m^2 [1]

(ii) the total length of the perimeter round the outer edges of the pools.

Answer _____ m [1]

The total length of the perimeter round the outer edges of the pools is 96 m.

(iii) Show that

$$y = 36 - \frac{3}{4}x$$

[2]

(iv) Find the dimensions of the pools which will give a maximum total area, showing that it is a maximum.

Answer Adults' pool _____ m by _____ m

Children's pool _____ m by _____ m [6]

THIS IS THE END OF THE QUESTION PAPER

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For Examiner's use only	
Question Number	**Marks**
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