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Rewarding Learning

General Certificate of Secondary Education
January 2015

Further Mathematics

Unit 2

Mechanics and Statistics

[GMF21]

MONDAY 19 JANUARY, MORNING

GMF21

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Complete in blue or black ink only. Do not write with a gel pen.
All working should be clearly shown since marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all thirteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

Formula Sheet

PURE MATHEMATICS
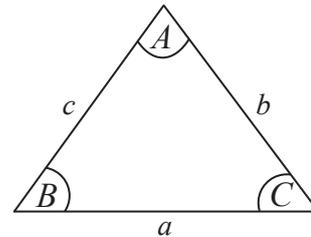
Quadratic equations: If $ax^2 + bx + c = 0$ $\qquad$ $(a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\qquad$ $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$

Differentiation: $\qquad$ If $y = ax^n$ $\qquad$ then $\qquad$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\qquad$ $\displaystyle\int ax^n \,\mathrm{d}x = \dfrac{ax^{n+1}}{n+1} + c$ $\qquad$ $(n \neq -1)$

Logarithms: $\qquad$ If $a^x = n$ $\qquad$ then $\qquad$ $x = \log_a n$

$\log(ab) = \log a + \log b$

$\log\left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: $\qquad$ If $\qquad$ $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\qquad$ $\det \mathbf{A} = ad - bc$

and $\qquad$ $\mathbf{A}^{-1} = \dfrac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\qquad$ $(ad - bc \neq 0)$

MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\dfrac{y}{x}\right)$

Uniform Acceleration:
$$v = u + at \qquad\qquad s = \tfrac{1}{2}(u + v)t$$
$$v^2 = u^2 + 2as \qquad\qquad s = ut + \tfrac{1}{2}at^2$$

where u is initial velocity t is time
     v is final velocity s is change in displacement
     a is acceleration

Newton's Second Law: $F = ma$

where F is resultant force m is mass
     a is acceleration

STATISTICS

Statistical measures: $\text{Mean} = \dfrac{\Sigma\, fx}{\Sigma\, f}$ $\text{Median} = L_1 + \dfrac{\left\{\dfrac{N}{2} - (\Sigma\, f)_1\right\} c}{f_{median}}$

where L_1 is lower class boundary of the median class
     N is total frequency
     $(\Sigma\, f)_1$ is the sum of the frequencies up to but not including the median class
     f_{median} is the frequency of the median class
     c is the width of the median class

$\text{Standard deviation} = \sqrt{\dfrac{\Sigma\, fx^2}{\Sigma\, f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability:
$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \dfrac{6\,\Sigma\, d^2}{n(n^2 - 1)}$$

9332   **3**   **[Turn over**

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Section A

Mechanics

You should spend approximately **one hour** on this section.

Take g = 10 m/s^2

1 (Throughout this question **i** and **j** denote unit vectors parallel to a set of standard *x-y* axes.)

A body is initially at the point with position vector $(2\mathbf{i} + \mathbf{j})$ m and moves with constant velocity $(4\mathbf{i} - 3\mathbf{j})$ m/s.

Calculate

(i) the speed of the body,

Answer ⎽⎽⎽⎽⎽⎽⎽⎽⎽⎽⎽⎽ m/s [2]

(ii) the position vector of the body after 5 seconds.

Answer ⎽⎽⎽⎽⎽⎽⎽⎽⎽⎽⎽⎽ m [3]

     [Turn over

Examiner Only	
Marks	Remark

2 A man walks at a constant speed of 2 m/s for 50 seconds. He stops for 20 seconds and then reverses direction and runs at a constant speed of 5 m/s for 30 seconds.

(i) On the axes below, sketch the displacement-time graph for the man.

Displacement (m)

Time (s)

[3]
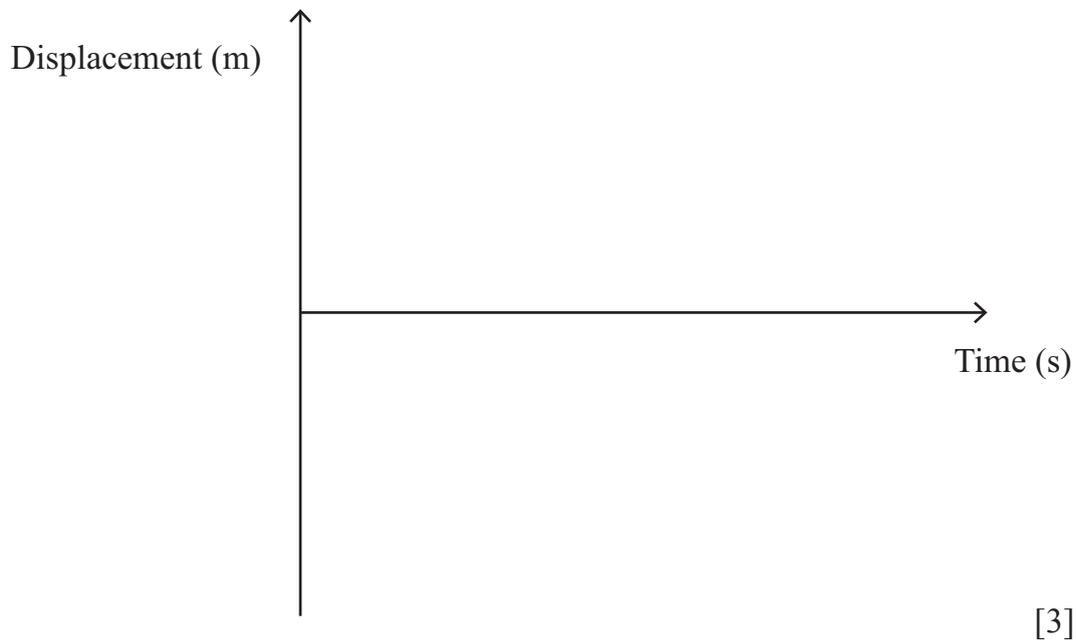
(ii) Calculate the **total distance** travelled by the man.

Answer _____ m [1]

(iii) Calculate the time from when the man starts to walk until he runs past his starting point.

Answer _____ s [2]

9332　　　　　　　　　　　　　　**7**　　　　　　　　　**[Turn over**

3   A train passes a point A, travelling at 45 km/h.
The train passes a point B 30 minutes later, travelling at 55 km/h.

Given that the train accelerates uniformly between A and B, calculate

**(i)**   the acceleration of the train between A and B, in km/h^2,

Answer _____ km/h^2 [2]

(ii)   the distance from A to B.

Answer _____ km [2]

9332   **8**

At the instant the train passes B, it starts to decelerate uniformly and comes to rest over a distance of 11 km.

Calculate

(iii) the deceleration of the train, in km/h^2,

Answer _____ km/h^2 [2]

(iv) the time taken for the train to come to rest after passing B, giving your answer in **minutes**.

Answer _____ minutes [2]

9332   **9**   **[Turn over**

4 Two blocks A and B, of masses 5 kg and 8 kg, are attached by a light inextensible string which passes over a smooth pulley. Block A is held at rest on a rough horizontal table and block B hangs in the air above horizontal ground, as shown in the diagram below.
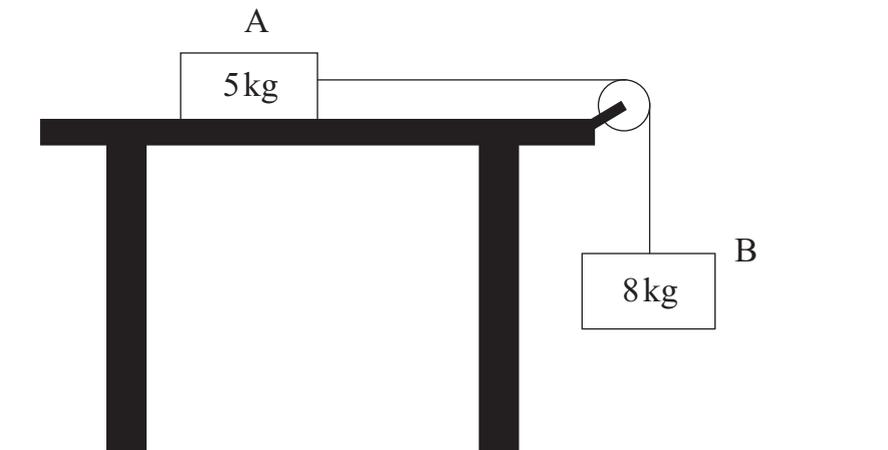
The system is released from rest. After 0.8 seconds block B has moved 1.6 m downwards. It is still above the ground and block A has not reached the pulley.

(i) On the diagram above, mark all the forces acting on each of the blocks.

[3]

(ii) Show that the acceleration of the blocks is $5 \, \text{m/s}^2$.

[2]

9332   **10**

(iii) Hence calculate the tension in the string.

Answer _____ N [2]

(iv) Calculate the coefficient of friction between the table and block A.

Answer _____ [3]

9332 **11** **[Turn over**

5 A uniform plank AB, of length 6 m and mass 15 kg, rests horizontally in equilibrium on two supports C and D, where AC = 2 m and BD = 1.5 m, as shown in the diagram below.

A   C   D   B

2 m   1.5 m

Calculate

(i) the magnitude of the reaction at D,

Answer _____ N [2]

(ii) the magnitude of the reaction at C.

Answer _____ N [1]

12

A block of mass M kg is placed at a point E on the plank, 0.5 m from B. The plank remains horizontal but is on the point of tilting about D.

(iii) State the reaction at C when the plank is on the point of tilting.

Answer _____ N [1]

(iv) Calculate the value of M

Answer _____ [2]

9332   **13**   **[Turn over**

The block is now removed from the point E and placed at a point F on the plank, x m from A. The plank remains horizontal but is now on the point of tilting about C.

(v) Find the value of x

Answer _____ [3]

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(Questions continue overleaf)

6 In a recent strongman competition, a competitor had to pull a large block of concrete, of mass 100 kg, along a rough horizontal runway. The competitor pulled the block with a rope inclined at an angle of 35° to the horizontal, as shown in the diagram below.
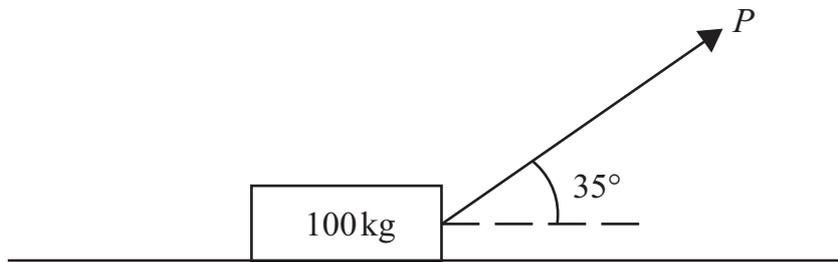
When the competitor applied a force *P* of 500 N, the block failed to move.

(i) Calculate the value of the frictional force at this time, giving your answer to the nearest newton.

Answer _____ N [2]

9332   **16**

When the competitor increased the force P to 600 N, the block was on the point of moving.

(ii) Calculate the value of the normal reaction now between the block and the runway, giving your answer to the nearest newton.

Answer ⎯⎯⎯⎯⎯⎯⎯⎯⎯ N [2]

(iii) Show that the coefficient of friction between the block and the runway is 0.75, to 2 decimal places.

[3]

9332   **17**   **[Turn over**

When the force P is increased to 700 N, the block begins to move.

Calculate

(iv) the new normal reaction between the block and the runway, giving your answer to the nearest newton,

Answer _____ N [2]

(v) the acceleration of the block.

Answer _____ m/s^2 [3]

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Section B

Statistics

You should spend approximately **one hour** on this section.

7   A geologist measured the masses of the rocks in his collection. The mean was calculated as 5.4 kg and the standard deviation as 2.6 kg.

The geologist later found out that he had recorded the mass of one of the rocks as 2.5 kg, whereas its true value was 5.2 kg.

He corrected this value and recalculated the mean and standard deviation for the rocks in his collection.

State whether the effect of the corrected value will be to increase, decrease or have no effect on

(i)   the mean,

Answer _____ [1]

(ii)   the standard deviation.

Answer _____ [1]

19

[Turn over

8 During a 24 hour period at a busy Accident and Emergency department the waiting time between arrival and either admission, transfer or discharge was recorded for each patient. The results are summarised in the table below.

Time (t) (hours)	$t \leqslant 1$	$1 < t \leqslant 2$	$2 < t \leqslant 3$	$3 < t \leqslant 4$	$4 < t \leqslant 5$	$5 < t \leqslant 6$
Number of patients	12	87	138	202	56	30

Calculate an estimate for the median waiting time for these patients.

Answer _____ hours [4]

20

9   A local golf club recorded the ages of its playing members. The results are summarised in the table below.

Age (years)	10–14	15–19	20–44	45–59	60–69	70–84
Number of playing members	30	24	90	165	124	81

Using the axes below, draw a histogram to represent this information.
Label each axis clearly.

[5]
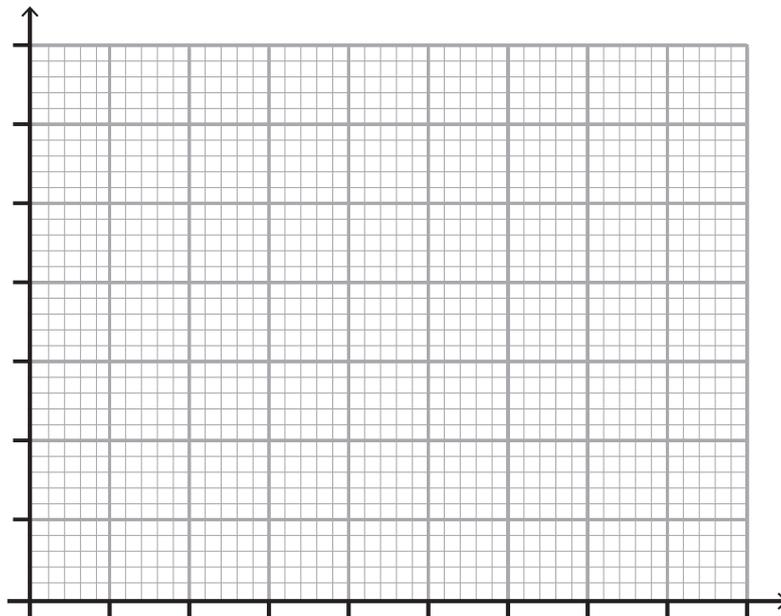
10 A youth orchestra consisting of 34 boys and 56 girls was on tour in Italy. On a day off from their concert tour the musicians were offered the choice of going sightseeing, going to the beach or remaining in the hotel.

The ratio of girls choosing each activity was 2:4:1 respectively.

A musician was selected at random to present a gift to the local mayor at that evening's concert.

(i) Calculate the probability that the musician selected was a girl who had chosen to stay in the hotel.

Answer _____ [3]

The probability that a musician chose to go to the beach is 0.6

(ii) Determine how many boys chose to go to the beach.

Answer _____ [3]

[Turn over

The same number of boys chose to go sightseeing as to stay in the hotel.

Two musicians were selected at random to give a vote of thanks after the concert.

(iii) Calculate the probability that both musicians selected were boys who had gone sightseeing.

Answer _____ [4]

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(Questions continue overleaf)

25 **[Turn over**

11 The mass, in kilograms, of each client attending a slimming class is recorded.
The table below shows these masses and the number of weeks each client has been attending the class.

Number of weeks	2	1	8	10	1	6	3	12	5	8
Mass (kg)	76	94	72	60	84	56	84	64	70	54

(i) Find the rank orders for the number of weeks and for the masses.

[2]

(ii) Calculate Spearman's coefficient of rank correlation.

Answer _____ [4]

(iii) Interpret your answer to part **(ii)**.

Answer _____ [1]

(iv) Calculate the mean number of weeks and the mean mass.

Answer _____

Answer _____ [1]

[Turn over

The data from the table are plotted on the graph below.

Examiner Only	
Marks	Remark

Mass (kg) vs **Number of weeks**
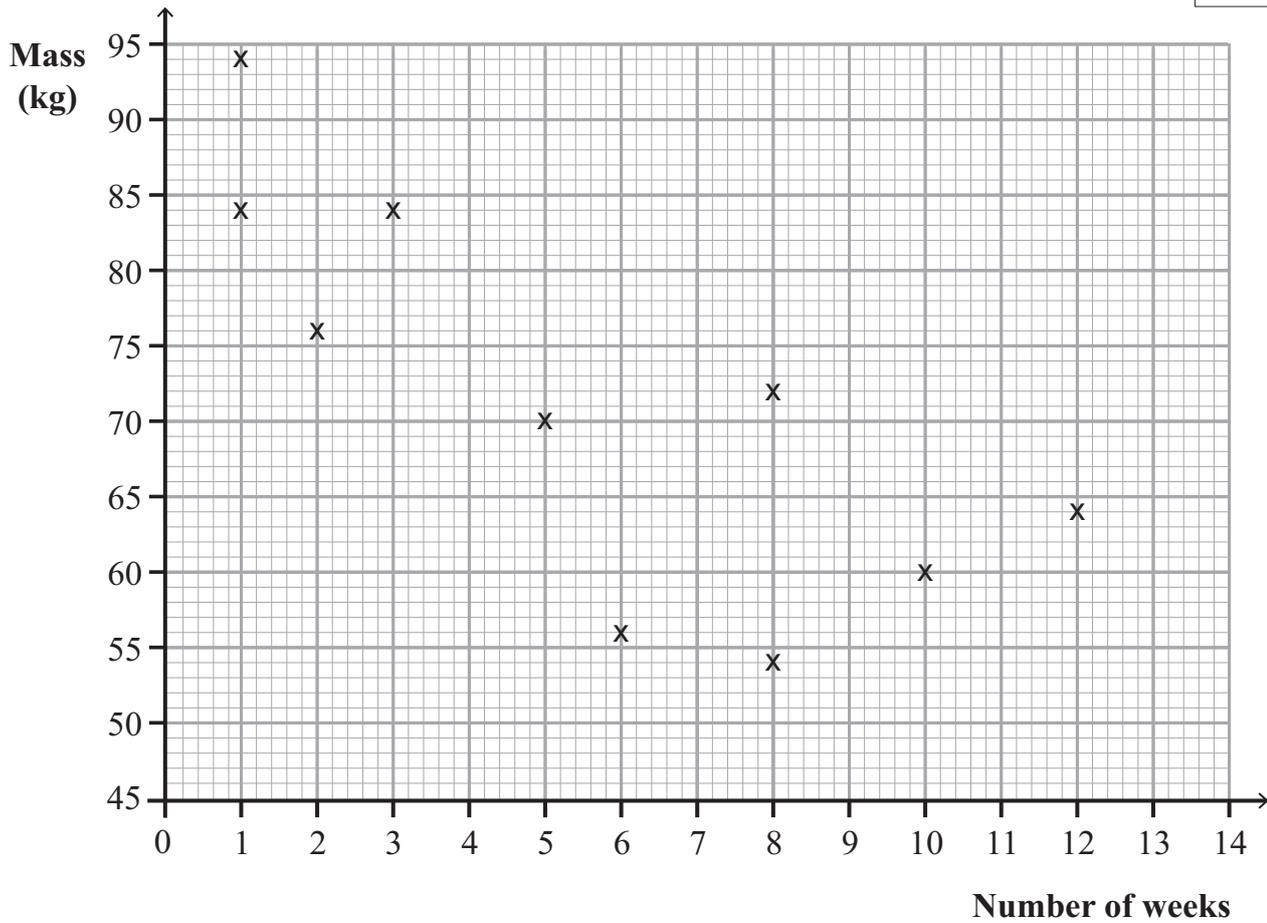
(v) Draw your line of best fit on the graph. [2]

(vi) Determine the equation of the line of best fit which you have drawn.

Answer _____ [3]

29

[Turn over

12 Kountry Kitchens offered built-in microwave ovens, boiling-water taps and coffee machines as extras with their new kitchen ranges.

Of the first 80 customers,
    64 chose a built-in microwave oven,
    24 chose a boiling-water tap,
    27 chose a coffee machine,
    17 chose a built-in microwave oven and a boiling-water tap,
    14 chose a boiling-water tap and a coffee machine,
    22 chose a built-in microwave oven and a coffee machine,
    8 did not choose any of these extras.

Let x be the number of customers who chose all three extras.

(i) Using a Venn diagram, determine the value of x

Answer _____ [4]

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30

(ii) Hence calculate the probability that a customer selected at random chose only a coffee machine.

Answer _____ [2]

31

[Turn over

13 The masses of apples in last year's harvest were analysed and gave the following results:

Lower quartile 115g;   Median 200g;   Upper quartile 255g.

An apple is selected at random from this harvest.

(i) What is the probability that it weighs less than 200 g?

Answer _____ [1]

(ii) What is the probability that it weighs more than 255 g?

Answer _____ [1]

(iii) Given that an apple weighs more than 115g, what is the probability that it weighs more than 255g?

Answer _____ [3]

An apple is selected at random from this harvest, weighed and replaced.
A second apple is then selected at random, weighed and replaced.

(iv) What is the probability that both weigh more than 115 g?

Answer _____ [2]

(v) What is the probability that one weighs more than 255 g and the other weighs less than 255 g?

Answer _____ [3]

THIS IS THE END OF THE QUESTION PAPER

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