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Rewarding Learning

General Certificate of Secondary Education
January 2015

Further Mathematics

Unit 1
Pure Mathematics

[GMF11]

FRIDAY 16 JANUARY, AFTERNOON

MARK SCHEME

GCSE Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M   indicates marks for correct method.

W   indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW   indicates marks for combined method and accurate working.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to regard candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

(a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;

(b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

1 (i) $A^2 = \begin{bmatrix} -2 & 3 \\ 5 & -4 \end{bmatrix}\begin{bmatrix} -2 & 3 \\ 5 & -4 \end{bmatrix} = \begin{bmatrix} 19 & -18 \\ -30 & 31 \end{bmatrix}$ M1 W1

(ii) $A^2 - 3B = \begin{bmatrix} 19 & -18 \\ -30 & 31 \end{bmatrix} - 3\begin{bmatrix} 1 & -7 \\ 4 & -2 \end{bmatrix}$

$\qquad = \begin{bmatrix} 19 & -18 \\ -30 & 31 \end{bmatrix} - \begin{bmatrix} 3 & -21 \\ 12 & -6 \end{bmatrix}$ MW1

$\qquad = \begin{bmatrix} 16 & 3 \\ -42 & 37 \end{bmatrix}$ W1   4

2 $x^2 = 14x + 2$
$x^2 - 14x - 2 = 0$ M1
$(x - 7)^2 - 49 - 2 = 0$ M1 MW1
$(x - 7)^2 - 51 = 0$
$(x - 7)^2 = 51$
$x - 7 = \pm\sqrt{51}$
$x = 7 \pm \sqrt{51}$ W1   4

3 $y = 2x^3 - \dfrac{1}{4x^5}$

$\quad = 2x^3 - \dfrac{1}{4}x^{-5}$

$\dfrac{dy}{dx} = 6x^2 + \dfrac{5}{4}x^{-6}$ MW1 MW1

$\dfrac{d^2y}{dx^2} = 12x - \dfrac{15}{2}x^{-7}$ **or** $12x - \dfrac{15}{2x^7}$ MW1 MW1   4

4 (i) $\sin\theta = 0.75$
$\quad \therefore \theta = 48.590°$ **or** $131.410°$
$\quad$ i.e. $\theta = 48.59°$ **or** $131.41°$ MW1 MW1

(ii) $\sin\left(\dfrac{x + 10°}{2}\right) = 0.75$

$\quad \therefore \dfrac{x + 10°}{2} = 48.590°$ **or** $131.410°$ M1

$\quad \therefore x = 87.18°$ **or** $252.82°$ W1 W1   5

AVAILABLE MARKS

5 **(i)** $P = \begin{bmatrix} 2 & -1 \\ 3 & 1 \end{bmatrix}$

$P^{-1} = \frac{1}{2+3} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$

$\qquad = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix}$ MW1 MW1

(ii) $Q = P^{-1} R$

$\qquad = \frac{1}{5} \begin{bmatrix} 1 & 1 \\ -3 & 2 \end{bmatrix} \begin{bmatrix} 13 & 4 \\ 7 & 6 \end{bmatrix}$ M2

$\qquad = \frac{1}{5} \begin{bmatrix} 20 & 10 \\ -25 & 0 \end{bmatrix}$ W1

$\qquad = \begin{bmatrix} 4 & 2 \\ -5 & 0 \end{bmatrix}$ 5

6 **(i)** $\frac{3x+1}{2x+1} \div \frac{1}{2x^2 - x - 1}$

$\qquad = \frac{3x+1}{2x+1} \div \frac{1}{(2x+1)(x-1)}$ MW1

$\qquad = \frac{3x+1}{2x+1} \times \frac{(2x+1)(x-1)}{1}$ M1

$\qquad = (3x+1)(x-1)$ MW1

(ii) $\frac{3x}{x-4} - \frac{2x+1}{x^2 - 16}$

$\qquad = \frac{3x}{x-4} - \frac{2x+1}{(x-4)(x+4)}$ M1

$\qquad = \frac{3x(x+4) - (2x+1)}{(x-4)(x+4)}$ M1 M1

$\qquad = \frac{3x^2 + 12x - 2x - 1}{(x-4)(x+4)}$

$\qquad = \frac{3x^2 + 10x - 1}{(x-4)(x+4)}$ W1   7

7 **(i)** **(a)** $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= 3\mathbf{p} + 12\mathbf{q}$ MW1

(b) $\overrightarrow{QS} = \overrightarrow{QR} + \overrightarrow{RS}$

$= \overrightarrow{QR} - \overrightarrow{QP}$

$= 12\mathbf{q} - 3\mathbf{p}$ MW1

(ii) $\overrightarrow{TR} = \overrightarrow{TS} + \overrightarrow{SR}$

$= -(-5\mathbf{p} + 20\mathbf{q}) + 3\mathbf{p}$ M1

$= 8\mathbf{p} - 20\mathbf{q}$ W1

(iii) $\overrightarrow{QT} = \overrightarrow{QS} + \overrightarrow{ST}$

$= (12\mathbf{q} - 3\mathbf{p}) + (-5\mathbf{p} + 20\mathbf{q})$

$= 32\mathbf{q} - 8\mathbf{p}$ MW1

so $\overrightarrow{QS} = \frac{3}{8} \overrightarrow{QT}$

$\therefore k = \frac{3}{8}$ W1 6

8 $4x + 2y - z = 16$ ①

     $5x - 3y - z = 3$ ②

     $14x + 2y - 3z = 40$ ③

     ② − ① → $x - 5y = -13$ ④   M1 W1

     $3 \times$ ② − ③ → $x - 11y = -31$ ⑤   M1 W1

     ④ − ⑤ → $6y = 18$ MW1

           $y = 3$ MW1

     From ④ $x = -13 + 15$ M1

         $x = 2$

     From ① $z = 8 + 6 - 16$

         $z = -2$ W1

     so $x = 2, y = 3, z = -2$ 8

9   **(a)** $\log_4 16 = 2x$

         $4^{2x} = 16$ M1

         $2x = 2$

         $x = 1$ W1

     (b)   **(i)** $\log_7 12 = \log_7 (6 \times 2)$

                   $= \log_7 6 + \log_7 2$

                   $= p + q$ MW1

        (ii) $\log_7 21 = \log_7 \left(\frac{7 \times 6}{2} \right)$ M1

                   $= \log_7 7 + \log_7 6 - \log_7 2$

                   $= 1 + q - p$ MW1   5

10 **(i)** $x - 2$ W1

(ii) $x + 6$ W1

(iii) $x : (x - 2) = (x + 6) : 8$ M1 W1

$\dfrac{x}{x - 2} = \dfrac{x + 6}{8}$ MW1

$8x = (x - 2)(x + 6)$

$8x = x^2 + 4x - 12$

$x^2 - 4x - 12 = 0$ MW1

$(x + 2)(x - 6) = 0$

$\therefore x = 6$ **or** $x = -2$ (impossible)

so 6 men in the Spinning class. W1 7

11 **(i)** The curve crosses the x-axis when

$2x^2 + 7x - 4 = 0$ M1

$(2x - 1)(x + 4) = 0$

$\therefore x = -4$ **or** $x = \dfrac{1}{2}$ W1

(ii) $\displaystyle\int_{-4}^{0} (2x^2 + 7x - 4)\, dx$ M1 W1

$= \left[\dfrac{2}{3} x^3 + \dfrac{7}{2} x^2 - 4x \right]_{-4}^{0}$ M1 W1

$= 0 - \left(-\dfrac{128}{3} + 56 + 16 \right)$

$= -29\dfrac{1}{3}$

so area $= 29\dfrac{1}{3}$ W1 7

12 (i) Gradient of tangent = –3   M1

                                                                          AVAILABLE MARKS

(ii) $\dfrac{dy}{dx} = 2x + 5$ MW1

so $2x + 5 = -3$ M1
$\quad\quad x = -4$
$\quad\quad y = -5$
so P is the point (–4, –5)   W1

(iii) Equation of tangent is
$y + 5 = -3\,(x + 4)$ MW1
$\quad\quad y = -3x - 17$ W1   6

13 (i) $X\hat{B}Y = 57.15° - 44.90°$
$\quad\quad\quad = 12.25°$ MW1

(ii) $\dfrac{BX}{\sin B\hat{Y}X} = \dfrac{XY}{\sin X\hat{B}Y}$

$BX = \dfrac{20 \sin 44.90°}{\sin 12.25°} = 66.536\,m \rightarrow 66.54\,m$ M1 W1

(iii) $AB^2 = AX^2 + BX^2 - 2.\,AX.\,BX \cos A\hat{X}B$

$AB^2 = 40^2 + 66.536^2 - 2 \times 40 \times 66.536 \cos 57.15°$ M1

$AB = 56.033\,m \rightarrow 56.03\,m$ W1

(iv) $\dfrac{\sin B\hat{A}X}{BX} = \dfrac{\sin A\hat{X}B}{AB}$
$\sin B\hat{A}X = \dfrac{66.536 \sin 57.15°}{56.033}$ M1

$B\hat{A}X = 85.999° \rightarrow 86.00°$ W1

(v) angle leaning from vertical

$= 90° - 86.00°$

$= 4.00°$ MW1   8

14 $2^{3x-2} = 5^{x-1}$

$\log 2^{3x-2} = \log 5^{x-1}$ M1

$(3x-2)\log 2 = (x-1)\log 5$ M1

$3x\log 2 - 2\log 2 = x\log 5 - \log 5$

$x(3\log 2 - \log 5) = 2\log 2 - \log 5$ M1 W1

$x = \dfrac{2\log 2 - \log 5}{3\log 2 - \log 5}$

$x = -0.47$ W1   5

15 **(i)** BC $= 2 - x - x = 2 - 2x$ MW1

(ii) A $= (x - 0.1)(2 - 2x)$ MW1 MW1

$\quad = 2.2x - 2x^2 - 0.2$

(iii) $\dfrac{dA}{dx} = 2.2 - 4x$

$\quad = 0$ MW1

$\therefore 4x = 2.2$

$x = 0.55$ W1

$\dfrac{d^2A}{dx^2} = -4 < 0 \; \therefore \; \text{maximum}$ M1

so maximum area when $x = 0.55$

(iv) Height of sides $= x = 0.55$

Width of base $= 2 - 2x = 0.9$ m   W1   7

16 **(i)** $4x - 2x^2 - x^3 = 0$

$x(4 - 2x - x^2) = 0$

$x = 0$ **or** $x = \dfrac{2 \pm \sqrt{20}}{-2}$

so coordinates are $(0, 0)$, $(1.2, 0)$ and $(-3.2, 0)$ W1 W1 W1

(ii) $\dfrac{dy}{dx} = 4 - 4x - 3x^2 = 0$ MW1 M1

$3x^2 + 4x - 4 = 0$

$(3x - 2)(x + 2) = 0$ M1

$x = \dfrac{2}{3}$ **or** $x = -2$

$y = 1\dfrac{13}{27}$ **or** -8 W2

so turning points at $\left(\dfrac{2}{3}, 1\dfrac{13}{27}\right)$ and $(-2, -8)$

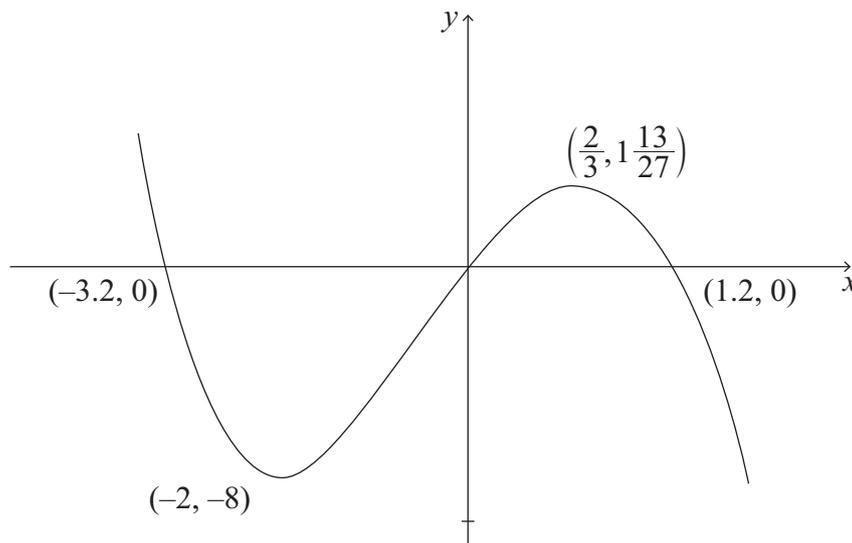**(iii)** $\dfrac{d^2y}{dx^2} = -4 - 6x$ MW1

at $x = \dfrac{2}{3}$, $\dfrac{d^2y}{dx^2} = -8$, $\therefore$ maximum at $\left(\dfrac{2}{3}, 1\dfrac{13}{27}\right)$

at $x = -2$, $\dfrac{d^2y}{dx^2} = 8$, $\therefore$ minimum at $(-2, -8)$ MW1

(iv)

M1 (shape)
W1 (points)   12

Total   **100**

AVAILABLE MARKS