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Rewarding Learning

General Certificate of Secondary Education
January 2019

Further Mathematics

Unit 2 **(With calculator)**
Mechanics and Statistics

[GMF21]

GMF21

WEDNESDAY 23 JANUARY, AFTERNOON

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all fourteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

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Formula Sheet

PURE MATHEMATICS

Quadratic equations: If $ax^2 + bx + c = 0$ $\qquad (a \neq 0)$
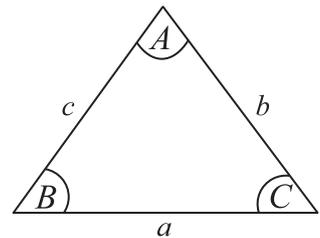
then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$

Differentiation: If $y = ax^n$ $\qquad$ then $\qquad \dfrac{dy}{dx} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \, dx = \dfrac{ax^{n+1}}{n+1} + c$ $\qquad (n \neq -1)$

Logarithms: If $a^x = n$ $\qquad$ then $\qquad x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\qquad \mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\qquad \det \mathbf{A} = ad - bc$

and $\qquad \mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\qquad (ad - bc \neq 0)$

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MECHANICS

Vectors: Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\dfrac{y}{x}\right)$

Uniform Acceleration: $v = u + at$ $s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$ $s = ut + \frac{1}{2}at^2$

where u is initial velocity t is time
            v is final velocity s is change in displacement
            a is acceleration

Newton's Second Law: $F = ma$

where F is resultant force m is mass
            a is acceleration

STATISTICS

Statistical measures: $\text{Mean} = \dfrac{\Sigma\, fx}{\Sigma\, f}$ $\text{Median} = L_1 + \dfrac{\left\{\dfrac{N}{2} - (\Sigma\, f)_1\right\} c}{f_{median}}$

where L_1 is lower class boundary of the median class
           N is total frequency
           $(\Sigma\, f)_1$ is the sum of the frequencies up to but not including the
                median class
           f_{median} is the frequency of the median class
           c is the width of the median class

$\text{Standard deviation} = \sqrt{\dfrac{\Sigma\, fx^2}{\Sigma\, f} - (\bar{x})^2}$ where $\bar{x}$ is the mean

Probability: $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$r = 1 - \dfrac{6\,\Sigma\, d^2}{n(n^2 - 1)}$

[Turn over

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Section A

Mechanics

You should spend approximately **one hour** on this section.

Take **g = 10 m/s^2** when required.

1 (Throughout this question **i** and **j** denote unit vectors parallel to a set of standard *x–y* axes.)

A block B of mass 2.5 kg is moving under the action of a constant force **F** newtons.

Initially the velocity of B is (−**i** − 13**j**) m/s and 4 seconds later the velocity of B is (−5**i** + 23**j**) m/s.

(i) Find, in terms of **i** and **j**, the acceleration of B.

Answer _____ m/s^2 [2]

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(ii) Calculate the **magnitude** of **F**.

Answer _____ N [3]

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2 A ball is thrown vertically upwards with an initial velocity of u m/s from a point P, which is 1.4 m above the ground.

The ball moves freely under gravity until it reaches the ground.

The greatest height attained by the ball is 14.2 m **above the ground**.

(i) Show that $u = 16$

[2]

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The ball reaches the ground t seconds after it has been thrown from P.

(ii) Find the value of t.

Answer _____ [4]

[Turn over

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3 A car accelerates uniformly from rest to a velocity of 18 m/s in T seconds.

The car then travels at a constant velocity of 18 m/s for $2T$ seconds.

It decelerates uniformly to rest in a further 40 seconds.

(i) Sketch a velocity–time graph to show the motion of the car.

Velocity
(m/s)

Time (s) [2]

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The total distance travelled by the car is 900 m.

Calculate

(ii) the value of *T*,

Answer _____ [3]

(iii) the initial acceleration of the car.

Answer _____ m/s^2 [1]

[Turn over

4   A uniform rod AB has mass 8 kg and length 4 m.

It rests in a horizontal position on two smooth supports placed at C and D, where AC = 0.6 m, as shown in the diagram below.

The reaction on the rod at C is 25 N.

(i) Calculate the magnitude of the reaction on the rod at D.

Answer _____ N [2]

(ii) Calculate the distance AD.

Answer _____ m [3]

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The support at D is now removed.

A mass of 30 kg is placed at A and a mass of M kg is placed at B, as shown in the diagram below.
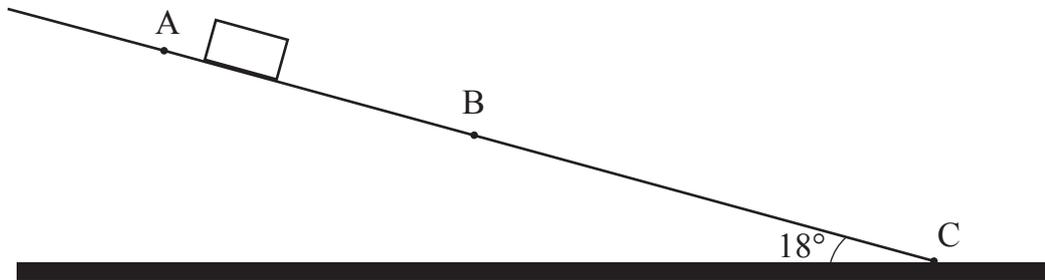
The uniform rod remains horizontal and in equilibrium.

(iii) Calculate the value of M.

Answer _____ [3]

5 A suitcase of mass 15 kg slides down a **rough** ramp, which is inclined at an angle of 18° to the horizontal, as shown in the diagram below.

(i) On the diagram above, mark all the forces acting on the suitcase. [2]

The points A, B and C are on the ramp, with AB = 4 m.

The suitcase passes A with velocity of 9 m/s and passes B with velocity of 7 m/s.

(ii) Calculate the **deceleration** of the suitcase.

Answer _____ m/s^2 [2]

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(iii) Calculate the coefficient of friction between the suitcase and the ramp.
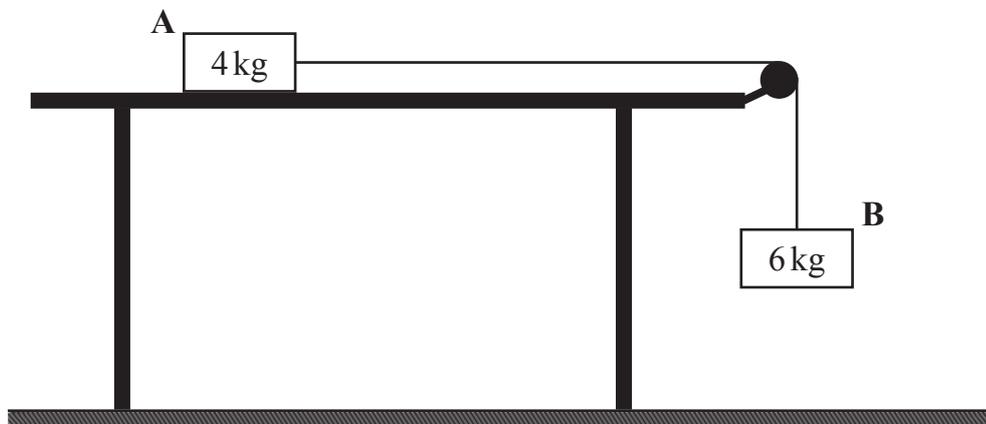
Answer _____ [5]

The suitcase comes to rest at C.

(iv) Calculate the distance AC.

Answer _____ m [2]

[Turn over

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32GMF2113

6 Two blocks A and B of masses 4 kg and 6 kg respectively are attached by a light inextensible string which passes over a smooth pulley. Block A is held at rest on a **rough** horizontal table and block B hangs freely above horizontal ground, as shown in the diagram below.

The coefficient of friction between the table and block A is 0.4

The blocks are released from rest.

(i) On the diagram above, mark all the forces acting on the blocks. [2]

(ii) Calculate the acceleration of the blocks.

Answer _____ m/s^2 [4]

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(iii) Calculate the tension in the string while the blocks are in motion.

Answer _____ N [2]

Once released, it takes block B 1.4 seconds to hit the ground.

(iv) Calculate the velocity with which block B is moving when it hits the ground.

Answer _____ m/s [2]

[Turn over

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Once block B hits the ground the string goes slack.

Block A stops before it reaches the pulley.

(v) Calculate the additional distance block A moves **after** block B hits the ground.

Answer _____ m [4]

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(Questions continue overleaf)

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Section B

Statistics

You should spend approximately **one hour** on this section.

7 The lengths, in centimetres, of 60 colouring pencils were measured and recorded to one decimal place.

The results are summarised in the table below.

Length (cm)	10.5–12.4	12.5–14.4	14.5–16.4	16.5–18.4
Number of pencils	15	16	18	11

Write down

(i) the limits of the median class,

Answer _____ [1]

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(ii) the boundaries of the modal class.

Answer _____ [1]

Each class has the same width.

(iii) Write down the class width.

Answer _____ cm [1]

[Turn over

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32GMF2119

8 Sean recorded, to the nearest minute, how long he spent on his exercise bike each day. His results are summarised in the table below.

| Time (minutes)	10–14	15–24	25–34	35–39	40–44	45–50
Number of days	2	7	26	15	6	4

Calculate an estimate for the median time.

Answer _____ minutes [4]

9   On Friday, a dental nurse recorded the ages of people visiting her surgery. The results are summarised in the table below.

Age (years)	0–9	10–14	15–24	25–39	40–54	55–64	65–
Number of people	6	12	14	30	24	14	0

Draw a histogram on the axes below to represent this information. Label both axes clearly.

[5]

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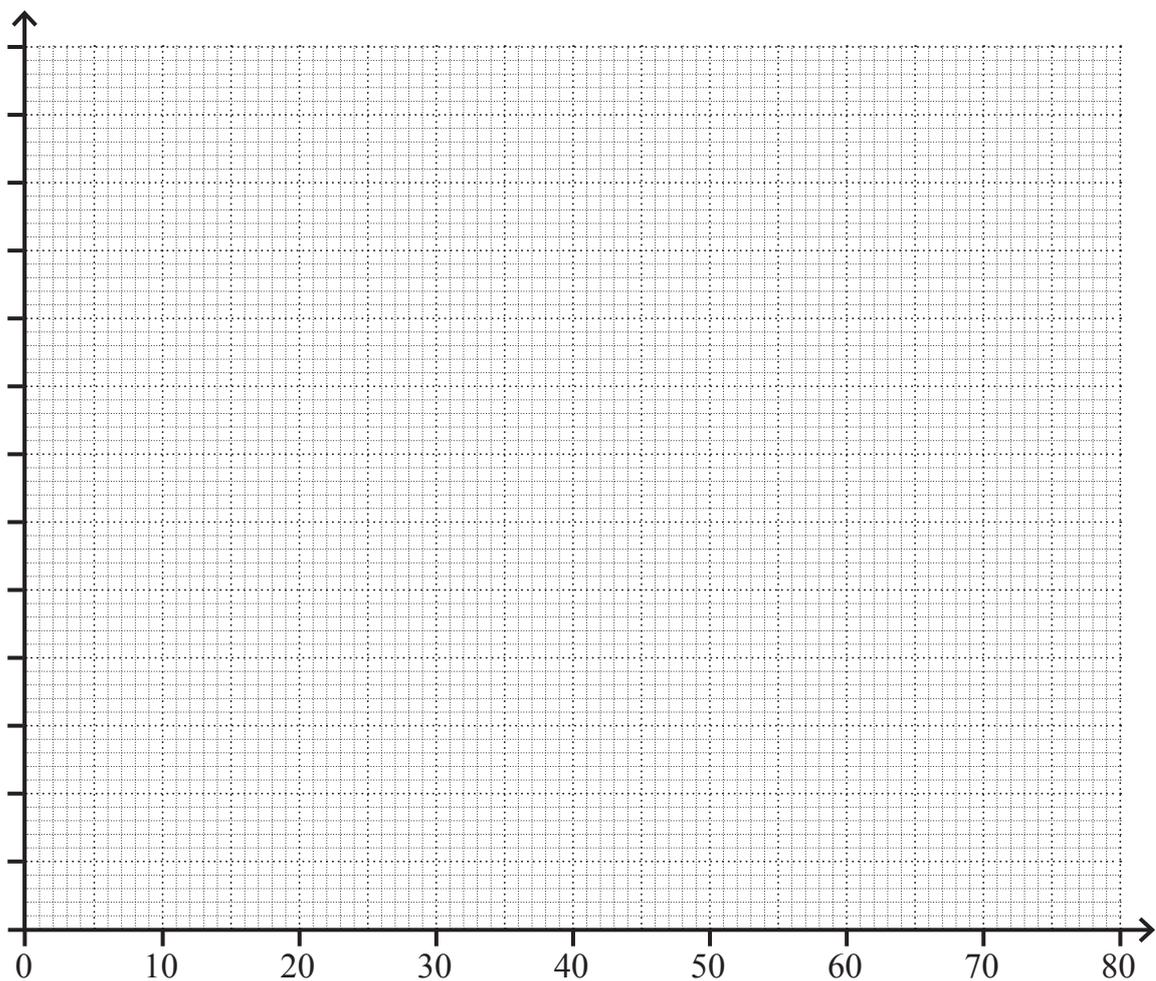
10 At a Christmas party 40 children were offered custard, jelly and ice cream for dessert.

   8 children chose all 3 items
13 children chose custard and jelly
15 children chose jelly and ice cream
11 children chose custard and ice cream
22 children chose custard
21 children chose ice cream
25 children chose jelly

(i) Illustrate this information on the Venn diagram below.

Custard   Jelly

Ice cream
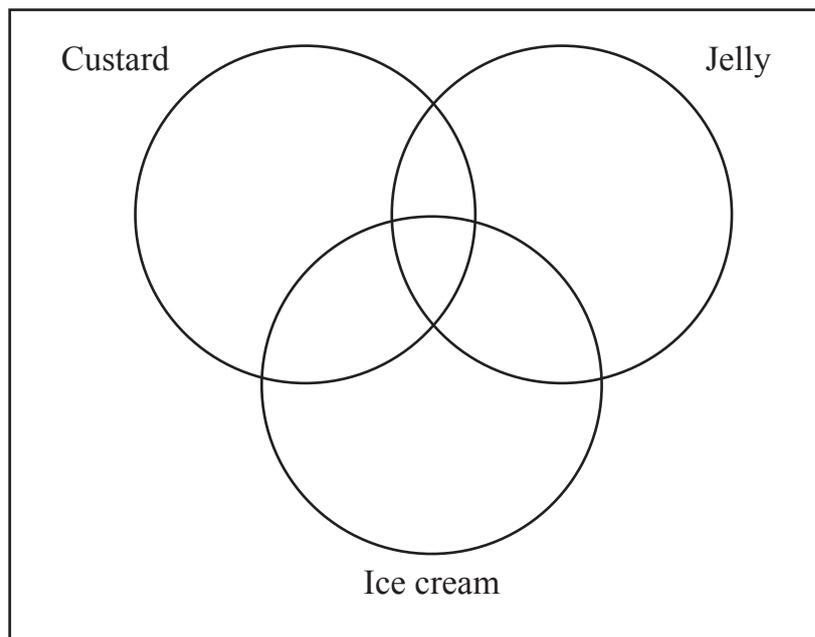
[3]

(ii) How many children did not have a dessert?

Answer _____ [1]

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(iii) What is the probability that a child selected at random chose **exactly** two items?

Answer _____ [2]

A child selected at random did **not** choose custard.

(iv) What is the probability that this child chose jelly?

Answer _____ [3]

[Turn over

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11 The table below shows the heights and masses of some dogs in Pete's Kennels one day.

Height (cm)	66	88	38	50	48	54	64	56	76
Mass (kg)	32	53	15	23	18	23	36	23	47
Ranks (Height)									
Ranks (Mass)									

(i) Write down the rank orders for the heights and masses in the table. [2]

(ii) Calculate Spearman's coefficient of rank correlation.

Answer _____ [4]

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(iii) Interpret your answer to part **(ii)**.

Answer _____ [1]

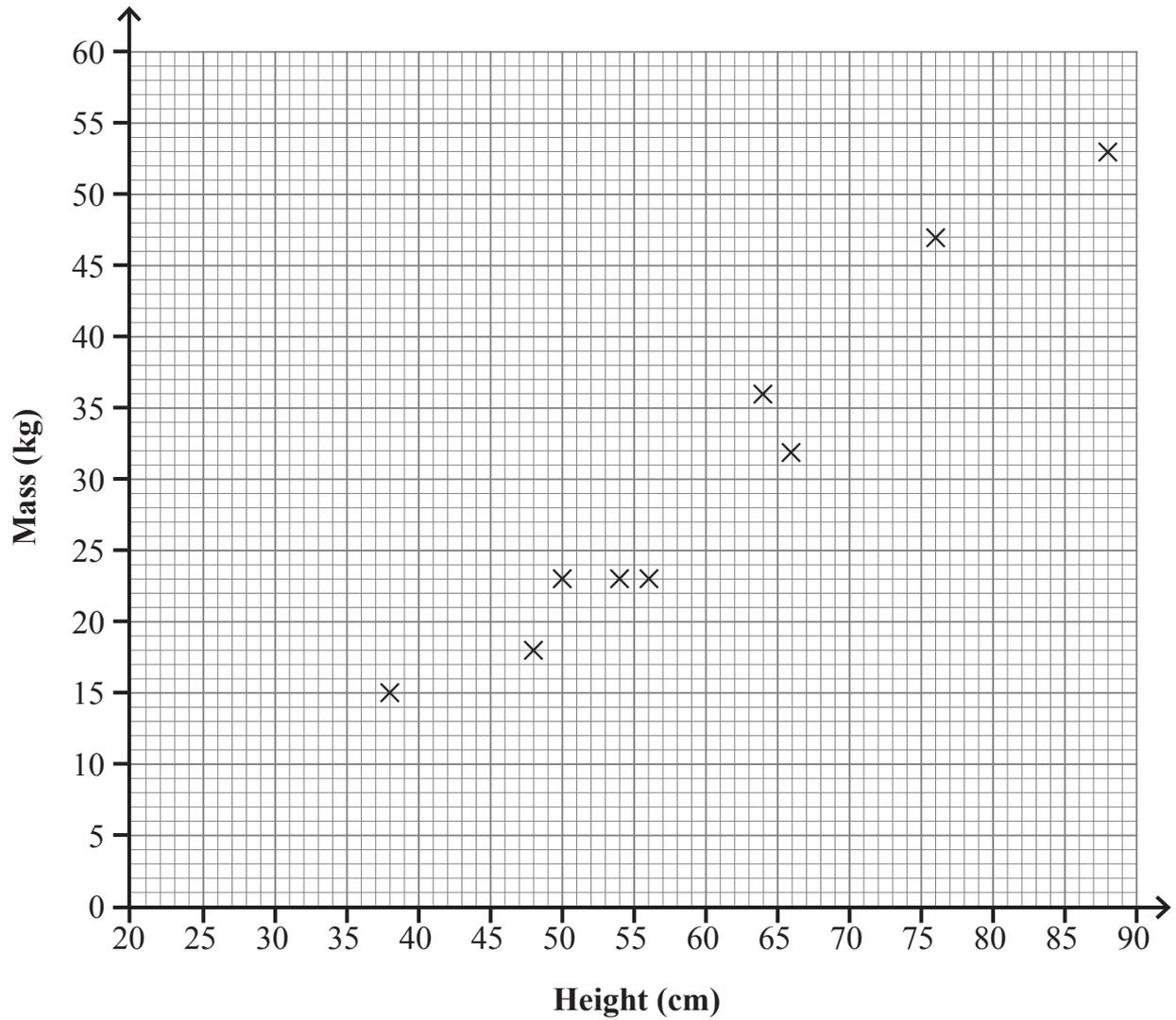**(iv)** Calculate the mean height and the mean mass.

Answer Mean height _____ cm

Mean mass _____ kg [1]

[Turn over

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The data from the table are plotted on the graph below.

(v) Draw your line of best fit on the graph. [2]

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(vi) Determine the equation of the line of best fit which you have drawn.

Answer _____ [3]

[Turn over

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12 A class of 8 girls and 12 boys took part in a penalty kick competition.

The mean number of goals scored by the girls was 6.5 and the standard deviation for the girls was 2.14

The mean number of goals scored by the boys was 5.5 and the standard deviation for the boys was 1.52

(i) Calculate the mean number of goals scored by the whole class.

You **must** show all your working.

Answer _____ [2]

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(ii) Calculate the standard deviation for the whole class.

You **must** show all your working.

Answer _____ [4]

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13 The masses of potatoes in a large box were analysed and gave the following results:

Lower quartile = 100 g   Median = 200 g   Upper quartile = 250 g

A potato is selected at random from the box.

(i) What is the probability that it weighs less than 200 g?

Answer _____ [1]

(ii) What is the probability that it weighs more than 250 g?

Answer _____ [1]

(iii) Given that a potato weighs more than 100 g, what is the probability that it weighs more than 250 g?

Answer _____ [3]

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14 Jack is given two bags, each containing 5 white balls and 2 black balls.

Jill is given one bag containing 10 white balls and 4 black balls.

Jack takes one ball from each of his bags and Jill takes two balls from her bag.

Who has the greater chance of picking two white balls?

You **must** show all your working.

Answer _____ [5]

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