



Rewarding Learning

**General Certificate of Secondary Education
2014**

Further Mathematics

Unit 2
Mechanics and Statistics

[GMF21]

FRIDAY 13 JUNE, MORNING

**MARK
SCHEME**

GCSE Further Mathematics

Introduction

The mark scheme normally provides the most popular solution to each question. Other solutions given by candidates are evaluated and credit given as appropriate; these alternative methods are not usually illustrated in the published mark scheme.

The marks awarded for each question are shown in the right hand column and they are prefixed by the letters **M**, **W** and **MW** as appropriate. The key to the mark scheme is given below:

M indicates marks for correct method.

W indicates marks for accurate working, whether in calculation, reading from tables, graphs or answers.

MW indicates marks for combined method and accurate working.

A later part of a question may require a candidate to use an answer obtained from an earlier part of the same question. A candidate who gets the wrong answer to the earlier part and goes on to the later part is naturally unaware that the wrong data is being used and is actually undertaking the solution of a parallel problem from the point at which the error occurred. If such a candidate continues to apply correct method, then the candidate's individual working must be **followed through** from the error. If no further errors are made, then the candidate is penalised only for the initial error. Solutions containing two or more working or transcription errors are treated in the same way. This process is usually referred to as "follow-through marking" and allows a candidate to gain credit for that part of a solution which follows a working or transcription error.

It should be noted that where an error trivialises a question, or changes the nature of the skills being tested, then as a general rule, it would be the case that not more than half the marks for that question or part of that question would be awarded; in some cases the error may be such that no marks would be awarded.

Positive marking:

It is our intention to regard candidates for any demonstration of relevant knowledge, skills or understanding. For this reason we adopt a policy of **following through** their answers, that is, having penalised a candidate for an error, we mark the succeeding parts of the question using the candidate's value or answers and award marks accordingly.

Some common examples of this occur in the following cases:

- (a) a numerical error in one entry in a table of values might lead to several answers being incorrect, but these might not be essentially separate errors;
- (b) readings taken from candidates' inaccurate graphs may not agree with the answers expected but might be consistent with the graphs drawn.

When the candidate misreads a question in such a way as to make the question easier, only a proportion of the marks will be available (based on the professional judgement of the examiner).

1 (i) $3(2\mathbf{i} + 3\mathbf{j}) - (4\mathbf{i} - 2\mathbf{j})$

$$= 6\mathbf{i} + 9\mathbf{j} - 4\mathbf{i} + 2\mathbf{j}$$

$$= 2\mathbf{i} + 11\mathbf{j}$$

M1

W1

(ii) $|2\mathbf{i} + 11\mathbf{j}| = \sqrt{2^2 + 11^2}$

$$= \sqrt{125} = 11.18$$

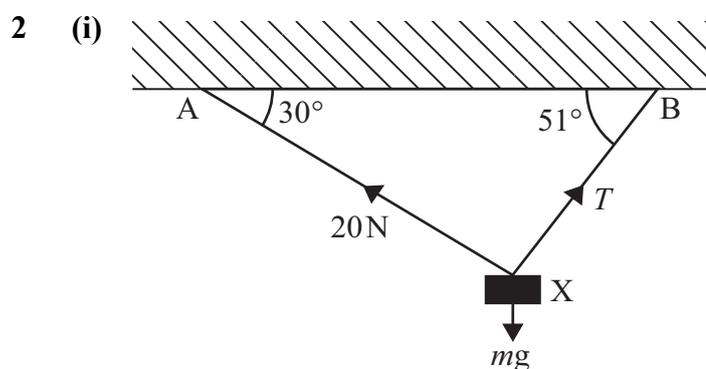
MW1

(iii) angle = $\tan^{-1}\left(\frac{11}{2}\right)$

$$= 79.70^\circ$$

MW1

4



W1

(ii) Resolve horizontally:

$$T \cos 51^\circ = 20 \cos 30^\circ$$

M1

$$T = \frac{20 \cos 30^\circ}{\cos 51^\circ} = 27.523 \text{ N} \rightarrow 27.52 \text{ N}$$

W1

(iii) Resolve vertically:

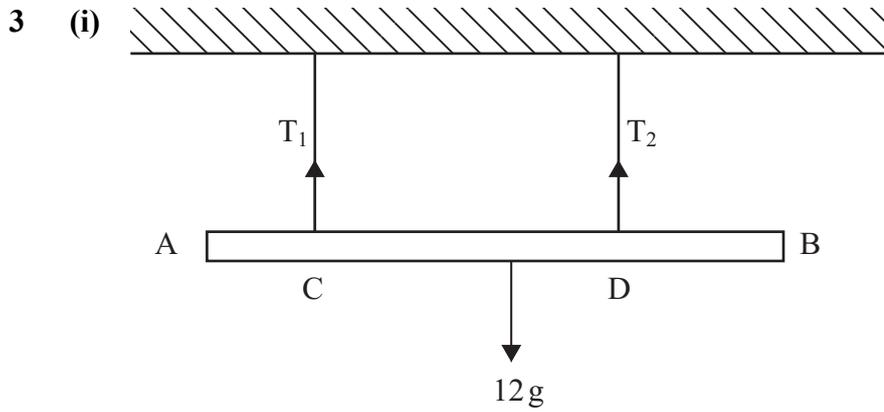
$$mg = 20 \sin 30^\circ + 27.523 \sin 51^\circ$$

M2

$$m = 3.14$$

W1

6



W1

(ii) Taking moments about C:

$$12g \times 2 = T_2 \times 3$$

MW1

$$\therefore T_2 = \frac{12g \times 2}{3} = 80\text{N}$$

W1

Resolve vertically:

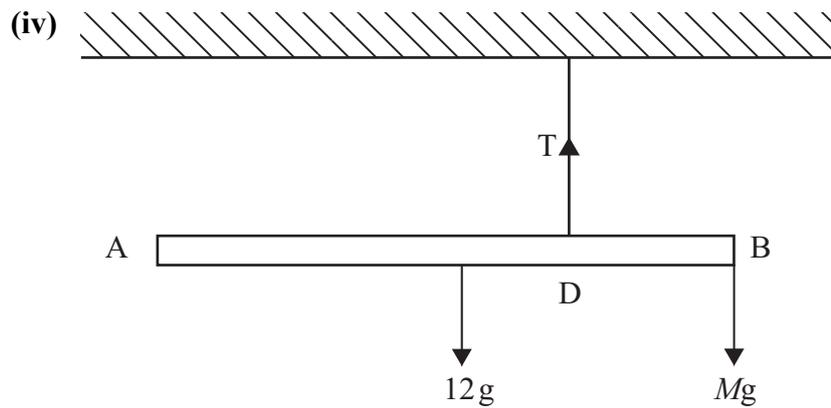
$$T_1 + T_2 = 12g$$

$$\therefore T_1 = 12g - 80 = 40\text{N}$$

MW1

(iii) Tension at C = 0 N

W1



Taking moments about D:

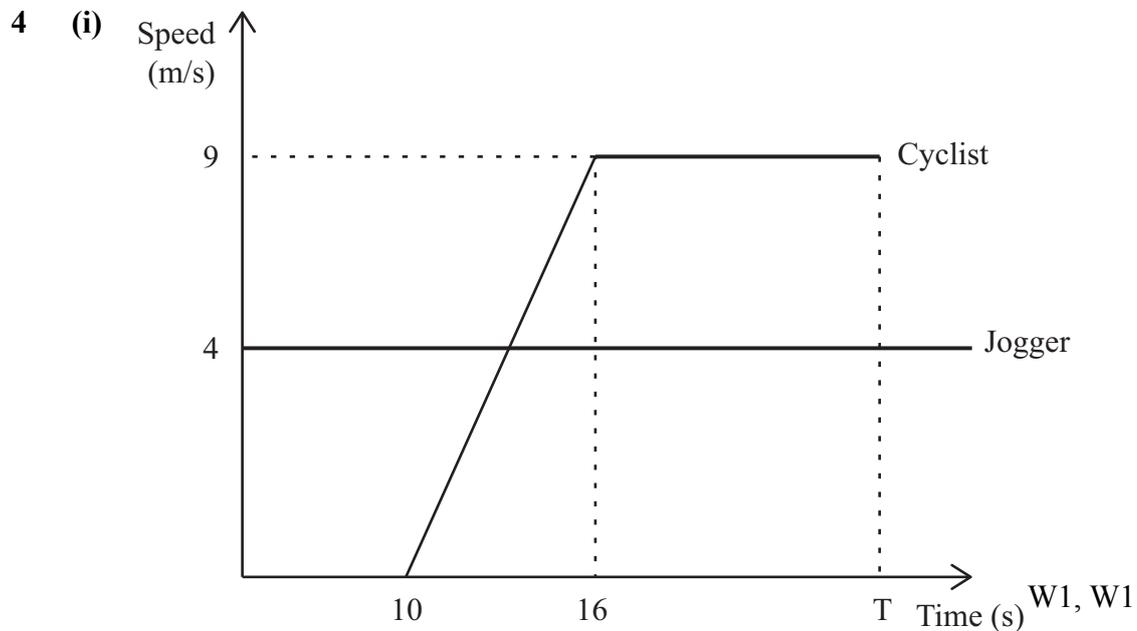
$$Mg \times 2 = 12g \times 1$$

MW1, MW1

$$\therefore M = 6$$

W1

8



(ii) Distance travelled by jogger = $4T$ MW1

$$\text{Distance travelled by cyclist} = 9(T - 16) + \frac{1}{2} \times 6 \times 9$$

$$= 9T - 117 \quad \text{MW1}$$

Distances equal, so

$$9T - 117 = 4T \quad \text{M1}$$

$$\therefore 5T = 117$$

$$\therefore T = 23.4 \text{ s} \quad \text{W1}$$

So time taken to reach jogger is $23.4 - 10$

$$= 13.4 \text{ s} \quad \text{MW1}$$

(iii) Distance travelled by cyclist is

$$9T - 117$$

$$= 9 \times 23.4 - 117 \quad \text{M1}$$

$$= 93.6 \text{ m} \quad \text{W1}$$

Alternative solution:

Distance travelled by cyclist = distance travelled by jogger

$$= 4 \times 23.4$$

$$= 93.6 \text{ m}$$

M1

W1

AVAILABLE
MARKS

9

5 (i) $s = ut + \frac{1}{2}at^2$

$$s = 0 \times 10 + \frac{1}{2} \times 5 \times 10^2$$

$$s = 250 \text{ m}$$

MW1

W1

(ii) $v = u + at$

$$v = 0 + 5 \times 10$$

$$v = 50 \text{ m/s}$$

MW1

W1

(iii) Reaches maximum height when speed = 0

$$v^2 = u^2 + 2as$$

$$0 = 50^2 + 2(-10)s$$

$$s = 125 \text{ m}$$

MW1

W1

$$\begin{aligned} \therefore \text{max height above ground} &= 250 + 125 \\ &= 375 \text{ m} \end{aligned}$$

W1

(iv) Time fuel burns = 10 s

Further time to reach max height:

$$v = u + at$$

$$0 = 50 + (-10)t$$

$$t = 5 \text{ s}$$

MW1

Time from max height to ground:

$$s = ut + \frac{1}{2}at^2$$

$$375 = 0 + \frac{1}{2} \times 10 \times t^2$$

$$t^2 = 75$$

$$t = 8.66 \text{ s}$$

MW1

W1

$$\text{Total time} = 10 + 5 + 8.66$$

$$= 23.66 \text{ s}$$

W1

AVAILABLE
MARKS

11

Alternative solution:

- (iv) Time fuel burns = 10 s
 Height when fuel burns out = 250 m
 Speed when fuel burns out = 50 m/s
 For remainder of motion

$$s = ut + \frac{1}{2} at^2$$

$$-250 = 50t - 5t^2$$

$$5t^2 - 50t - 250 = 0$$

$$t^2 - 10t - 50 = 0$$

$$t = \frac{10 \pm \sqrt{100 + 200}}{2}$$

$$= -3.66 \text{ s (ignore) or } 13.66 \text{ s}$$

$$\text{Total time} = 10 + 13.66$$

$$= 23.66 \text{ s}$$

MW1, MW1

W1

W1

AVAILABLE
MARKS

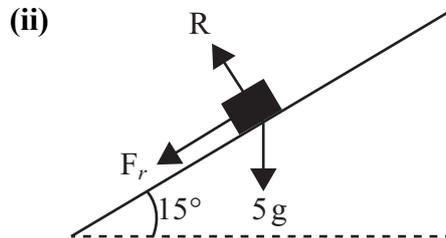
6 (i) $v = u + at$

$$0 = 8 + 2a$$

$$a = -4$$

$$\therefore \text{deceleration} = 4 \text{ m/s}^2$$

MW1



Resolve perpendicular to plane:

$$R = 5g \cos 15^\circ$$

$$R = 48.296 \text{ N} \rightarrow 48.30 \text{ N}$$

MW1

(iii) Resolve parallel to plane:

$$F_r + 5g \sin 15^\circ = ma = 5 \times 4$$

MW1, MW1

$$F_r = 20 - 5g \sin 15^\circ$$

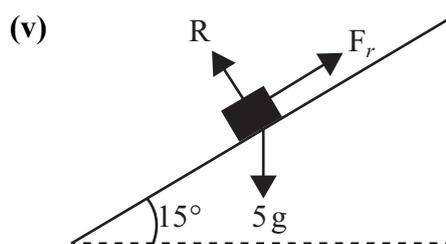
$$F_r = 7.059 \text{ N} \rightarrow 7.06 \text{ N}$$

W1

(iv) $F_r = \mu R$

$$\mu = \frac{7.059}{48.296} = 0.15$$

M1, W1



$$\text{Accelerating force} = 5g \sin 15^\circ - F_r$$

M1, M2

$$= 5.882 \text{ N}$$

Accelerating force = mass \times acceleration

$$5.882 = 5a$$

M1

$$a = 1.18 \text{ m/s}^2$$

W1

12

7 (i) Times in seconds are:

58, 137, 50, 64, 81, 58, 52, 116, 71, 133

$$\Sigma x = 820$$

$$\therefore \text{mean} = \frac{820}{10} = 82 \text{ seconds}$$

$$= 1 \text{ minute } 22 \text{ seconds}$$

M1

W1

(ii) $\Sigma x^2 = 77544$

$$\therefore \text{s.d.} = \sqrt{\frac{77544}{10} - 82^2}$$

$$= 32.10 \text{ s}$$

M1

W1

4

8 (i) median class is 91–100

$$\text{median} = 90.5 + \frac{(30.5 - 29) \times 10}{23}$$

$$= 91.15 \text{ m}$$

MW1 (90.5+)

MW1 (30.5–29)

MW1 (10/23)

W1 (answer)

(ii) Using class groups assumes equal distribution within class

M1

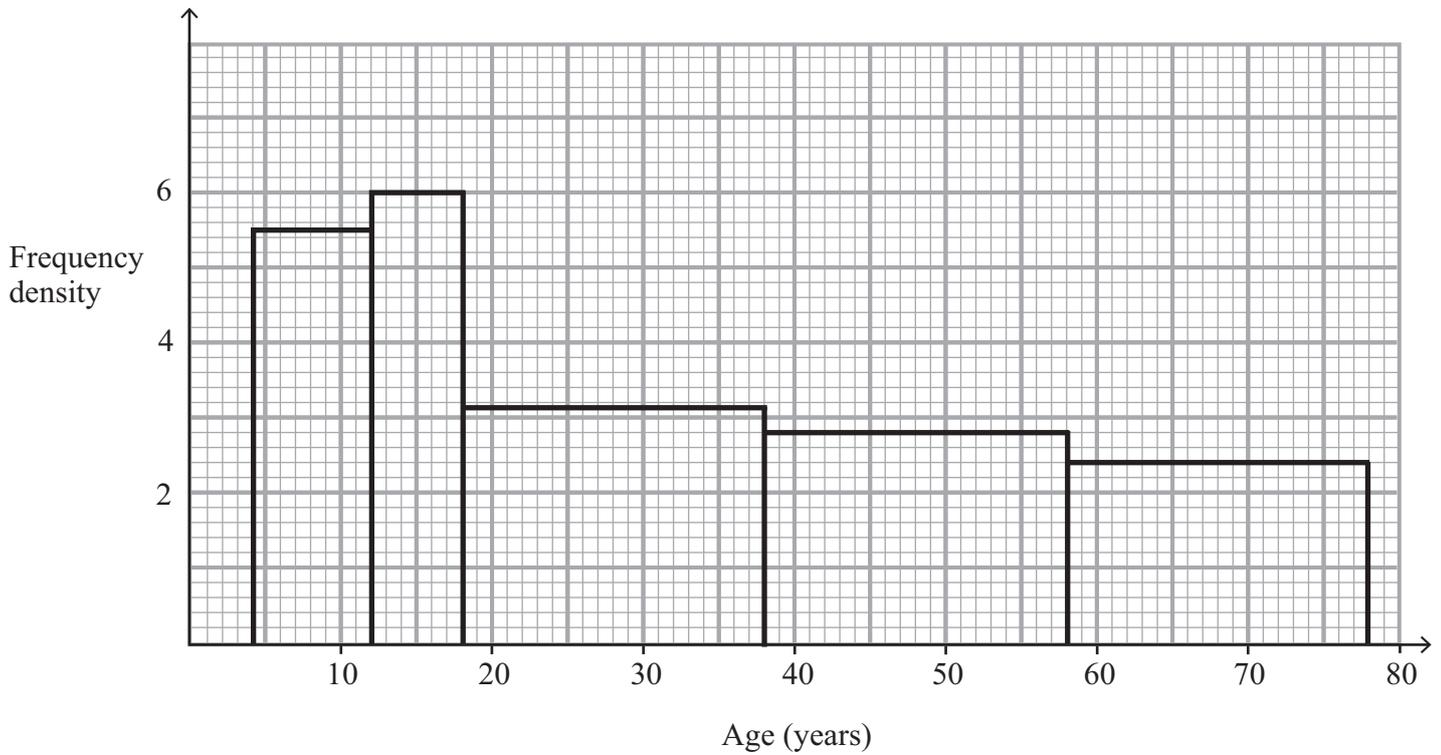
5

9

Age (years)	4–11	12–17	18–37	38–57	58–77
Number of visitors	44	36	63	56	48
Frequency density	5.5	6	3.15	2.8	2.4

M1, W1

AVAILABLE MARKS



MW1 (labels)
 MW1 (boundaries)
 MW1 (heights)

5

10 (i) Cannot get total of 6, so

$$P(6) = 0 \quad \text{M1}$$

(ii) A score of 8 is obtained by throwing a 6 followed by a 2, so

$$P(8) = \frac{1}{6} \times \frac{1}{6} = \frac{1}{36} \quad \text{M1, W1}$$

(iii) A score > 8 is obtained by throwing a 6 followed by a 3, 4, 5 or 6, so

$$P(> 8) = \frac{1}{6} \times \frac{4}{6} = \frac{1}{9} \quad \text{M2, W1}$$

$$(iv) P(12 | > 8) = \frac{P(12)}{P(> 8)} = \frac{\frac{1}{36}}{\frac{1}{9}} \quad \text{MW1, MW1}$$

$$= \frac{1}{4} \quad \text{W1}$$

Alternative solution:

One way to get 12; 4 ways to get > 8

M1, M1

$$\therefore P(12 | > 8) = \frac{1}{4}$$

W1

11 (i) There are $x + 4$ sweets in the bag.

$$\text{So } P(\text{red}) = \frac{4}{x + 4} \quad \text{MW1}$$

$$(ii) P(\text{red, red}) = \frac{4}{x + 4} \times \frac{3}{x + 3} \quad \text{M1, W1}$$

$$= \frac{12}{(x + 4)(x + 3)}$$

$$(iii) \frac{12}{(x + 4)(x + 3)} = \frac{1}{6} \quad \text{MW1}$$

$$x^2 + 7x + 12 = 72$$

$$x^2 + 7x - 60 = 0$$

$$(x + 12)(x - 5) = 0$$

$$\therefore x = -12 \text{ (impossible) or } x = 5$$

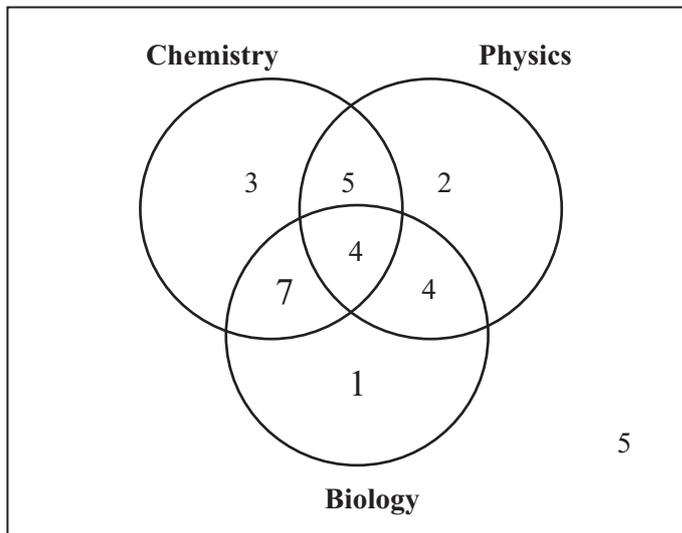
i.e. 5 yellow sweets W1

AVAILABLE
MARKS

9

5

12 (i)



3 × MW1

(ii) Revised no subjects on $(31 - 3 - 5 - 2 - 7 - 4 - 4 - 1)$ days

$$= 5 \text{ days}$$

W1

(iii) Revised 2 subjects on $(7 + 4 + 5)$ days = 16 days

$$\text{So } P(\text{exactly 2}) = \frac{16}{31}$$

MW1, MW1

(iv) Did not revise Physics on 16 days

MW1

Revised Chemistry but not Physics on $(3 + 7)$ days = 10 days

MW1

$$\text{So } P(\text{Chemistry/no Physics}) = \frac{10}{16} = \frac{5}{8}$$

MW1

9

AVAILABLE
MARKS

13 (i)

Ranks (First piece)	7	2	5	3	1	10	4	9	8	6
Ranks (Second piece)	7.5	5	7.5	6	1	9	3.5	10	3.5	2

or

Ranks (First piece)	4	9	6	8	10	1	7	2	3	5
Ranks (Second piece)	3.5	6	3.5	5	10	2	7.5	1	7.5	9

MW1, MW1

(ii)

d^2	0.25	9	6.25	9	0	1	0.25	1	20.25	16
-------	------	---	------	---	---	---	------	---	-------	----

M1, W1

$$r = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

$$= 1 - \frac{6(63)}{10(99)} = 0.62$$

M1, W1

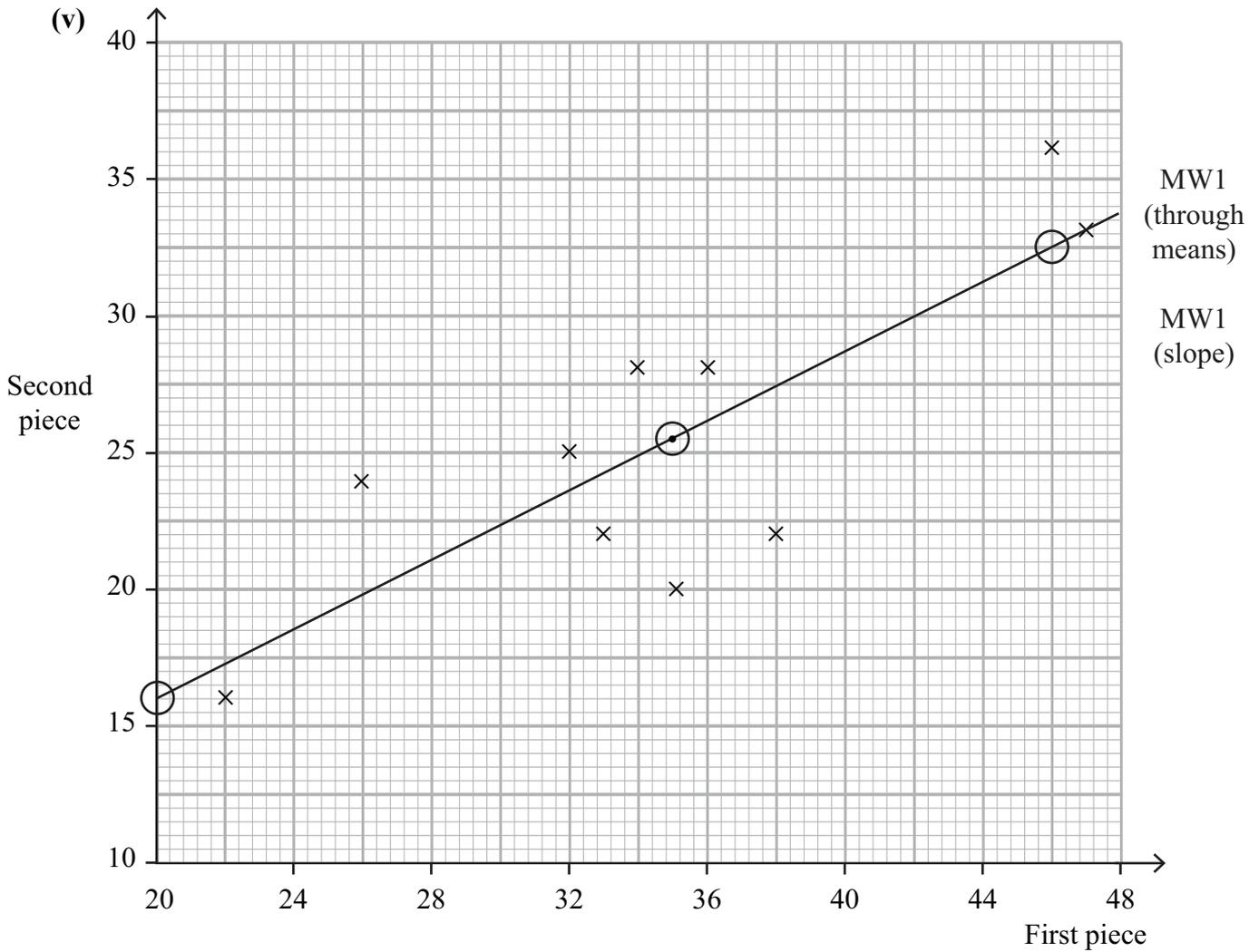
(iii) Positive correlation

M1

(iv) First piece: mean = $\frac{349}{10} = 34.9$ Second piece: mean = $\frac{254}{10} = 25.4$

MW1

AVAILABLE
MARKS



(vi) Gradient = $\frac{32.5 - 16.0}{46 - 20} = 0.63$

Using means

$$25.4 = 0.63(34.9) + c$$

$$\therefore c = 3.41$$

So equation is

$$y = 0.63x + 3.41$$

M1

M1

MW1

Total

AVAILABLE MARKS

13

100