Rewarding Learning

General Certificate of Secondary Education
2016

Further Mathematics

Unit 1

Pure Mathematics

[GMF11]

THURSDAY 16 JUNE, AFTERNOON

MARK SCHEME

General Marking Instructions

Introduction

Mark schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of students in schools and colleges.

The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes, therefore, are regarded as part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

		AVAILABLE MARKS
1 **(i)** $\mathbf{A} - 3\mathbf{C} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - 3\begin{bmatrix} 4 \\ -2 \end{bmatrix}$		
$= \begin{bmatrix} 2 \\ 3 \end{bmatrix} - \begin{bmatrix} 12 \\ -6 \end{bmatrix} = \begin{bmatrix} -10 \\ 9 \end{bmatrix}$	MW1, W1	
(ii) $\mathbf{AB} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} [-1 \quad 2]$		
$= \begin{bmatrix} -2 & 4 \\ -3 & 6 \end{bmatrix}$	M1 (2 × 2 matrix) W1	4
2 **(i)** $x^2 - 4x - 5$		
$= (x-2)^2 - 4 - 5$	M1 (for $(x-2)^2$)	
$= (x-2)^2 - 9$	W1	
(ii) **(a)** min value $= -9$	MW1	
(b) value of $x = 2$	MW1	4
3 $y = \frac{2}{3}x^3 - \frac{7}{x} = \frac{2}{3}x^3 - 7x^{-1}$		
(i) $\frac{dy}{dx} = 2x^2 + 7x^{-2}$ or $2x^2 + \frac{7}{x^2}$	MW1, MW1	
(ii) $\frac{d^2y}{dx^2} = 4x - 14x^{-3}$ or $4x - \frac{14}{x^3}$	MW1, MW1	4

		AVAILABLE MARKS
4 $\int_1^2 \left(x^3 - \frac{3}{2x^2}\right) dx$		
$= \int_1^2 \left(x^3 - \frac{3}{2}x^{-2}\right) dx$		
$= \left[\frac{x^4}{4} + \frac{3}{2}x^{-1}\right]_1^2$	MW1, MW1	
$= \left[\frac{x^4}{4} + \frac{3}{2x}\right]_1^2$		
$= \left[4 + \frac{3}{4}\right] - \left[\frac{1}{4} + \frac{3}{2}\right]$	M1	
$= \frac{19}{4} - \frac{7}{4}$		
$= 3$	W1	4

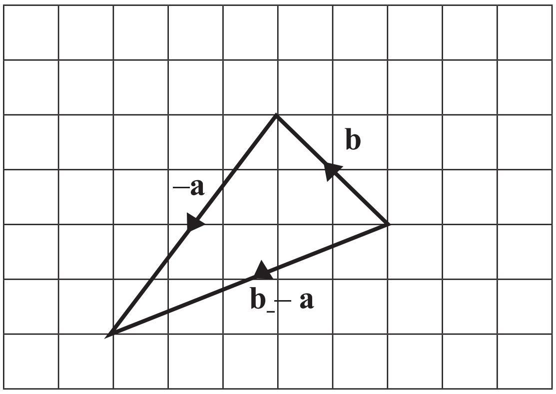
AVAILABLE MARKS

5 **(a)** $p\begin{bmatrix}5\\2\end{bmatrix} + 3\begin{bmatrix}-1\\q\end{bmatrix} = \begin{bmatrix}7\\-5\end{bmatrix}$

So $5p - 3 = 7 \therefore p = 2$ MW1

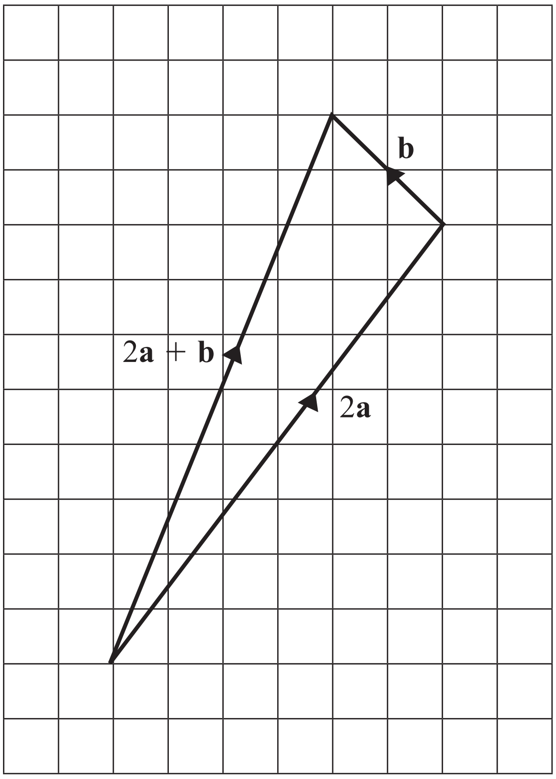
$4 + 3q = -5 \therefore q = -3$ MW1

(b) **(i)** $\mathbf{b} - \mathbf{a}$



MW1

(ii) $2\mathbf{a} + \mathbf{b}$



MW1 4

		AVAILABLE MARKS
6 **(i)** $\cos\theta = -0.65$		
$\theta = 130.542°$ or $229.458°$		
$\rightarrow$ $130.54°$ or $229.46°$	MW1, MW1	
(ii) $2x + 20 = 130.542°$ or $229.458°$	M1	
$2x = 110.542°$ or $209.458°$		
$x = 55.27°$ or $104.73°$	W1, W1	5
7 $3^{2x+1} = 8^{4-x}$		
$\log(3^{2x+1}) = \log(8^{4-x})$	M1	
$(2x+1)\log 3 = (4-x)\log 8$ or $2x + 1 = (4-x)\log_3 8$	MW1	
$2x\log 3 + \log 3 = 4\log 8 - x\log 8$ or $2x + 1 = 4\log_3 8 - x\log_3 8$	W1	
$2x\log 3 + x\log 8 = 4\log 8 - \log 3$ or $2x + x\log_3 8 = 4\log_3 8 - 1$		
$x = \dfrac{4\log 8 - \log 3}{2\log 3 + \log 8}$ or $x = \dfrac{4\log_3 8 - 1}{2 + \log_3 8}$	MW1	
$= 1.69$	W1	5

		AVAILABLE MARKS
8 **(i)** Det $\mathbf{P} = (4 \times (-2)) - (1 \times 3) = -11$		
$\mathbf{P}^{-1} = -\frac{1}{11}\begin{bmatrix} -2 & -3 \\ -1 & 4 \end{bmatrix}$ or $\frac{1}{11}\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$	MW1, MW1	
(ii) $\begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 34 \\ 3 \end{bmatrix}$	M1	
$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 1 & -2 \end{bmatrix}^{-1}\begin{bmatrix} 34 \\ 3 \end{bmatrix}$	M2	
$= \frac{1}{11}\begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}\begin{bmatrix} 34 \\ 3 \end{bmatrix}$		
$= \frac{1}{11}\begin{bmatrix} 77 \\ 22 \end{bmatrix}$		
$= \begin{bmatrix} 7 \\ 2 \end{bmatrix}$		
$x = 7, y = 2$	W1	6

		AVAILABLE MARKS
9 **(i)** $\dfrac{x}{x^2 + 6x + 8} + \dfrac{1}{x + 2}$		
$= \dfrac{x}{(x + 4)(x + 2)} + \dfrac{1}{x + 2}$	MW1	
$= \dfrac{x + (x + 4)}{(x + 4)(x + 2)}$	MW1	
$= \dfrac{2x + 4}{(x + 4)(x + 2)}$		
$= \dfrac{2(x + 2)}{(x + 4)(x + 2)}$		
$= \dfrac{2}{x + 4}$	W1	
(ii) $\dfrac{x^2 - 9}{4x + 12} \div \dfrac{x^2 + x - 12}{6}$		
$\dfrac{(x - 3)(x + 3)}{4(x + 3)} \div \dfrac{(x + 4)(x - 3)}{6}$	MW1, MW1	
$= \dfrac{(x - 3)(x + 3)}{4(x + 3)} \times \dfrac{6}{(x + 4)(x - 3)}$	M1	
$= \dfrac{3}{2(x + 4)}$	W1	7

AVAILABLE MARKS

10 **(i)** $y = 0$ when $x = -3$ or $x = 4$

Crosses x-axis at (–3,0) and (4,0) MW1, MW1

(ii) $y = (x + 3)(x - 4) = x^2 - x - 12$ MW1

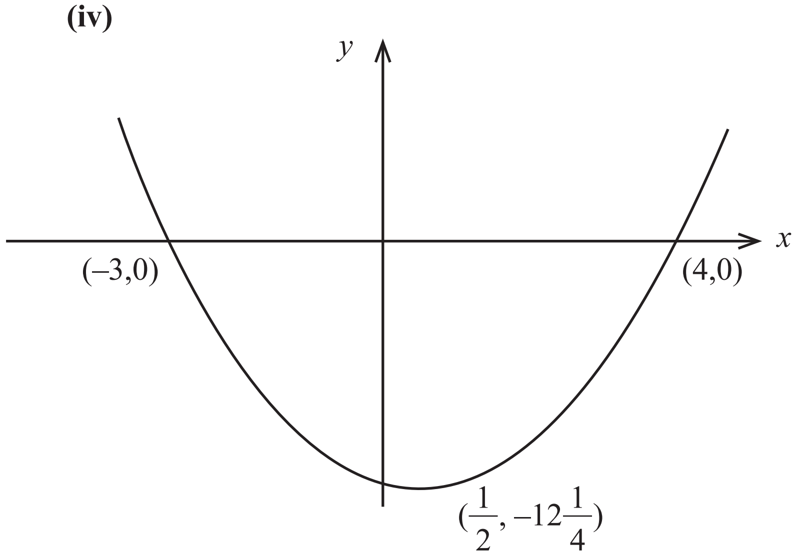
$\frac{dy}{dx} = 2x - 1 = 0$ MW1, M1

$\therefore \quad x = \frac{1}{2} \quad y = -12\frac{1}{4}$ W1

Turning point at $\left(\frac{1}{2}, -12\frac{1}{4}\right)$

(iii) $\frac{d^2y}{dx^2} = 2 > 0, \therefore$ minimum point MW1

(iv)



M1 (curve)
MW1 (points) 9

				AVAILABLE MARKS
11 **(i)**	$60x + 84y + 48z = 3696 \quad (\div 12)$		MW1	
	$\therefore \quad 5x + 7y + 4z = 308$			
(ii)	$56x + 63y + 42z = 3045 \quad (\div 7)$		MW1	
	$\therefore \quad 8x + 9y + 6z = 435$			
(iii)	$45(x - 5) + 54(y - 5) + 18(z - 5) = 1746$		M1	
	$45x - 225 + 54y - 270 + 18z - 90 = 1746$			
	$45x + 54y + 18z = 2331 \quad (\div 9)$		W1	
	$\therefore \quad 5x + 6y + 2z = 259$			
(iv)	$5x + 7y + 4z = 308$	①		
	$8x + 9y + 6z = 435$	②		
	$5x + 6y + 2z = 259$	③		
	① – ③ → $y + 2z = 49$	④	M1, W1	
	8 × ③ – 5 × ② → $3y - 14z = -103$	⑤	M1, W1	
	7 × ④ + ⑤ → $10y = 240$		M1	
	$y = 24$		W1	
	From ④ $2z = 49 - 24$			
	$z = 12.5$			
	From ① $5x = 308 - 168 - 50$			
	$x = 18$		M1	
	Stalls £18 Main circle £24 Balcony £12.50		W1	12

AVAILABLE MARKS

12 **(i)** $XZ^2 = XY^2 + YZ^2 - 2.XY.YZ\cos X\hat{Y}Z$

$20^2 = 30^2 + 40^2 - 2(30)(40)\cos X\hat{Y}Z$ M1

$\cos X\hat{Y}Z = \frac{30^2 + 40^2 - 20^2}{2 \times 30 \times 40}$ $\left(= \frac{2100}{2400} = \frac{7}{8} = 0.875\right)$ M1

$X\hat{Y}Z = 28.955° \rightarrow 28.96°$ W1

(ii) area $= \frac{1}{2}XY.YZ\sin X\hat{Y}Z$

$= \frac{1}{2} \times 30 \times 40\sin 28.955°$ M1

$= 290.47\text{ km}^2$ W1

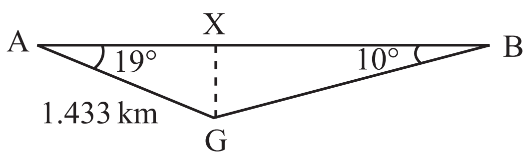
(iii) $A\hat{G}B = 180° - 19° - 10° = 151°$ MW1

(iv) $\frac{AG}{\sin A\hat{B}G} = \frac{AB}{\sin A\hat{G}B}$

$AG = \frac{4\sin 10°}{\sin 151°}$ M1

$= 1.433\text{ km} \rightarrow 1.43\text{ km}$ W1

(v)



Distance ship has to travel is AX

$\frac{AX}{AG} = \cos 19°$

$AX = 1.433\cos 19°$ M1

$= 1.35\text{ km}$ W1 10

		AVAILABLE MARKS
13 $\dfrac{dy}{dx} = 2x + \dfrac{3}{2}$	MW1	
At $x = -1$, $\dfrac{dy}{dx} = -2 + \dfrac{3}{2} = -\dfrac{1}{2}$		
Gradient of tangent $= -\dfrac{1}{2}$	MW1	
Gradient of normal at A = 2	MW1	
$y = 2x + c$		
$2 = 2(-1) + c$		
$c = 4$		
Equation of normal is $y = 2x + 4$	MW1	4

			AVAILABLE MARKS
14	**(i)** Time for first stage $= \frac{140}{x}$	MW1	
	Time for second stage $= \frac{x}{64}$	MW1	
	(ii) Total time $= \frac{140}{x} + \frac{x}{64} = 3$	M1	
	$8960 + x^2 = 192x$	MW1	
	$x^2 - 192x + 8960 = 0$		
	(iii) $x = \frac{192 \pm \sqrt{192^2 - 4 \times 8960}}{2}$	MW1	
	[or $(x - 112)(x - 80) = 0$]		
	$x = 80$ or 112		
	$x = 80$ as x must be less than 96	W1	
	(iv) Total distance $= 140 + 80 = 220$ km	MW1	7

		AVAILABLE MARKS
15 (i) (a) $\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$	MW1	
$\overrightarrow{OM} = \overrightarrow{OA} + \frac{1}{2}\overrightarrow{AB}$		
$= \mathbf{a} + \frac{1}{2}(-\mathbf{a} + \mathbf{b})$		
$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$	MW1	
(b) $\overrightarrow{BC} = \overrightarrow{BO} + \overrightarrow{OC}$		
$= -\mathbf{b} + 2\mathbf{a} + \mathbf{b}$		
$= 2\mathbf{a}$	MW1	
$\overrightarrow{ON} = \overrightarrow{OB} + \frac{1}{2}\overrightarrow{BC}$		
$= \mathbf{b} + \mathbf{a}$	MW1	
(ii) $\overrightarrow{OA} = \mathbf{a}$		
$\overrightarrow{BN} = \frac{1}{2}\overrightarrow{BC} = \mathbf{a}$	M1, MW1	
[**Alternative solution**		
$\overrightarrow{OB} = \mathbf{b}$		
$\overrightarrow{AN} = \overrightarrow{AO} + \overrightarrow{ON} = -\mathbf{a} + \mathbf{a} + \mathbf{b} = \mathbf{b}$	M1, MW1]	
Opposite sides are equal and parallel, so OANB is a parallelogram.		6

			AVAILABLE MARKS
16 **(i)** Income	$I = x(1000 - 2x)$		
	$= 1000x - 2x^2$ pounds	MW1	
(ii) Total cost	$C = 20\,000 + 400x$		
Profit	$P = I - C$		
	$= (1000x - 2x^2) - (20\,000 + 400x)$	M1, M1, MW1	
	$= 600x - 2x^2 - 20\,000$		
(iii) $\frac{dP}{dx} = 600 - 4x$		MW1	
$= 0$ for maximum		M1	
$x = 150$		W1	
$\frac{d^2P}{dx^2} = -4 < 0, \therefore$ maximum		MW1	
(iv) Cost per member	$= 1000 - 2x$		
	$= 1000 - 300$		
	$= £700$	MW1	9
		Total	**100**