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Rewarding Learning

General Certificate of Secondary Education
2017

Further Mathematics

Unit 1 **(With calculator)**

Pure Mathematics

[GMF11]

GMF11

TUESDAY 13 JUNE, MORNING

TIME

2 hours.

INSTRUCTIONS TO CANDIDATES

Write your Centre Number and Candidate Number in the spaces provided at the top of this page.
You must answer the questions in the spaces provided.
Do not write outside the boxed area on each page, on blank pages or tracing paper.
Complete in black ink only. **Do not write with a gel pen.**
All working should be clearly shown in the spaces provided. Marks may be awarded for partially correct solutions.
Where rounding is necessary give answers correct to **2 decimal places** unless stated otherwise.
Answer **all sixteen** questions.

INFORMATION FOR CANDIDATES

The total mark for this paper is 100.
Figures in brackets printed down the right-hand side of pages indicate the marks awarded to each question or part question.
You may use a calculator.
The Formula Sheet is on pages 2 and 3.

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Formula Sheet

PURE MATHEMATICS

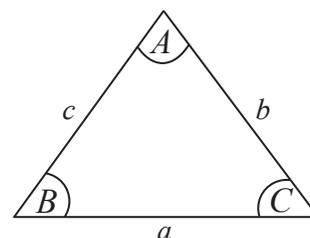Quadratic equations: If $ax^2 + bx + c = 0$ $\quad\quad (a \neq 0)$

then $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Trigonometry: $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$

$a^2 = b^2 + c^2 - 2bc \cos A$

Area of triangle $= \frac{1}{2} ab \sin C$

Differentiation: If $y = ax^n$ $\quad$ then $\quad$ $\dfrac{\mathrm{d}y}{\mathrm{d}x} = nax^{n-1}$

Integration: $\displaystyle\int ax^n \, \mathrm{d}x = \dfrac{ax^{n+1}}{n+1} + c$ $\quad\quad (n \neq -1)$

Logarithms: If $a^x = n$ $\quad$ then $\quad$ $x = \log_a n$

$\log (ab) = \log a + \log b$

$\log \left(\dfrac{a}{b}\right) = \log a - \log b$

$\log a^n = n \log a$

Matrices: If $\quad$ $\mathbf{A} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

then $\quad$ $\det \mathbf{A} = ad - bc$

and $\quad$ $\mathbf{A}^{-1} = \dfrac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$ $\quad\quad (ad - bc \neq 0)$

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MECHANICS

Vectors:

Magnitude of $x\mathbf{i} + y\mathbf{j}$ is given by $\sqrt{x^2 + y^2}$

Angle between $x\mathbf{i} + y\mathbf{j}$ and $\mathbf{i}$ is given by $\tan^{-1}\left(\dfrac{y}{x}\right)$

Uniform Acceleration:

$$v = u + at \qquad\qquad s = \tfrac{1}{2}(u + v)t$$
$$v^2 = u^2 + 2as \qquad\qquad s = ut + \tfrac{1}{2}at^2$$

where u is initial velocity t is time
              v is final velocity s is change in displacement
              a is acceleration

Newton's Second Law: $F = ma$

where F is resultant force m is mass
              a is acceleration

STATISTICS

Statistical measures: $\text{Mean} = \dfrac{\Sigma\, fx}{\Sigma\, f}$ $\text{Median} = L_1 + \dfrac{\left\{\frac{N}{2} - (\Sigma\, f)_1\right\}c}{f_{median}}$

where L_1 is lower class boundary of the median class
              N is total frequency
              $(\Sigma\, f)_1$ is the sum of the frequencies up to but not including the median class
              f_{median} is the frequency of the median class
              c is the width of the median class

$$\text{Standard deviation} = \sqrt{\dfrac{\Sigma\, fx^2}{\Sigma\, f} - (\bar{x})^2} \qquad \text{where } \bar{x} \text{ is the mean}$$

Probability:

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$$

Bivariate Analysis: Spearman's coefficient of rank correlation is given by

$$r = 1 - \dfrac{6\,\Sigma\, d^2}{n(n^2 - 1)}$$

[Turn over

1 Matrices **P** and **Q** are defined by

$$\mathbf{P} = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \quad \text{and} \quad \mathbf{Q} = \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$$

Express as a single matrix:

(i) 2**P** – **Q**

Answer _____ [1]

(ii) **P**^{2}

Answer _____ [2]

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(iii) Hence find the matrix **R** such that

$$\tfrac{1}{2}\mathbf{R} + \mathbf{Q} = \mathbf{P}^2$$

Answer _____ [2]

[Turn over

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2 (i) Rewrite the equation

$$x + \frac{23}{x} = 10$$

in the form $ax^2 + bx + c = 0$

Answer _____ [1]

(ii) Hence solve the equation

$$x + \frac{23}{x} = 10$$

by completing the square. Give your answer in the form $p \pm \sqrt{q}$, where p and q are whole numbers.

Answer _____ [4]

[Turn over

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3 If $y = 3x - \dfrac{1}{2x^3}$ find $\dfrac{d^2y}{dx^2}$

Answer _____ [3]

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4 Find $\displaystyle\int \frac{2}{3x^9}\,\mathrm{d}x$

Answer _____ [2]

[Turn over

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5 **(a)** The vectors **a**, **b** and **c** are defined by

$$\mathbf{a} = \begin{bmatrix} 2 \\ 3 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} \quad \text{and} \quad \mathbf{c} = \begin{bmatrix} 4 \\ -2 \end{bmatrix}$$

Find the vector **x** if

$$2\mathbf{x} + \mathbf{c} = \mathbf{a} - 4\mathbf{b}$$

Answer _____ [2]

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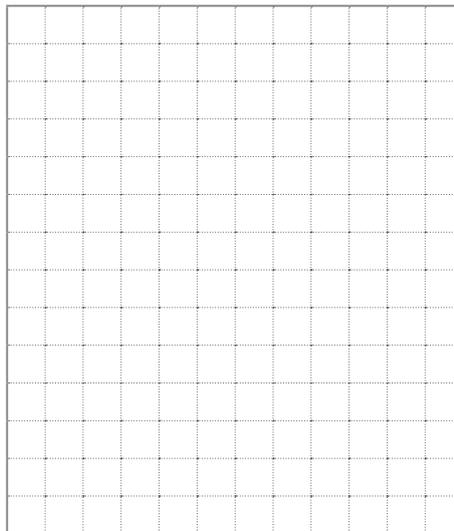**(b)** The vectors **p** and **q** are shown in the diagram below.

p **q**
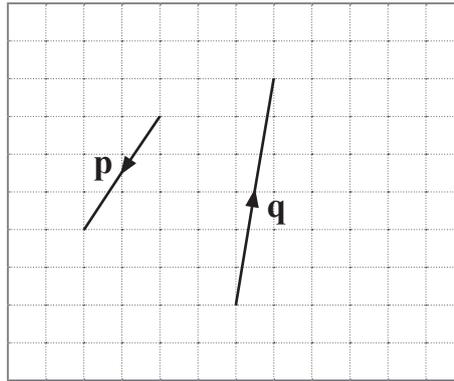
On the grids below, draw diagrams to show the vectors

(i) **p** + **q**

[1]

(ii) **q** − 2**p**

[1]

[Turn over

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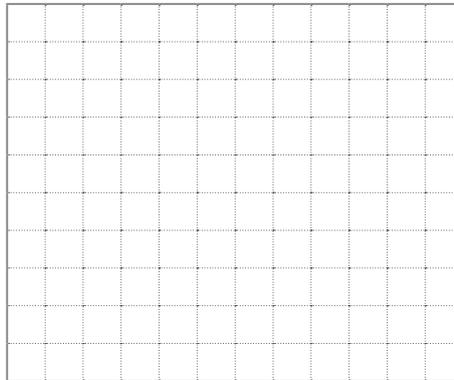
6 (i) Solve the equation

$$\tan \theta = 1.56$$

for $0° \leqslant \theta \leqslant 360°$

Answer _____ [2]

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(ii) Hence solve the equation

$$\tan\left(\frac{5}{6}x - 45°\right) = 1.56$$

for $0° \leqslant x \leqslant 360°$

Answer _____ [3]

[Turn over

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7 Matrices **A** and **B** are defined by

$$\mathbf{A} = \begin{bmatrix} 7 & 6 \\ 3 & 4 \end{bmatrix} \quad \text{and} \quad \mathbf{B} = \begin{bmatrix} 19 & 33 \\ 11 & 17 \end{bmatrix}$$

Find the matrix **X** such that **AX** = **B**

Answer _____ [5]

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(Questions continue overleaf)

8 **(i)** Show that

$$\frac{3(x-4)}{x^2-16} + \frac{2}{x-1}$$

simplifies to

$$\frac{5(x+1)}{(x+4)(x-1)}$$

[4]

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(ii) Hence solve the equation

$$\frac{3(x-4)}{x^2-16} + \frac{2}{x-1} = \frac{5}{6}$$

Answer _____ [3]

[Turn over

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9 (i) Express

$$2 \log 3x + \log 2 - \log 7$$

as a single log.

Answer _____ [3]

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(ii) Hence solve the equation

$$2 \log_5 3x + \log_5 2 - \log_5 7 = 2$$

Answer _____ [2]

[Turn over

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10 An octagonal commemorative plaque is formed by cutting four equal isosceles triangles from the corners of a square of side x cm.

The isosceles triangles are right-angled and the two equal sides of each triangle are of length $(x - 9)$ cm, as shown in the diagram below.
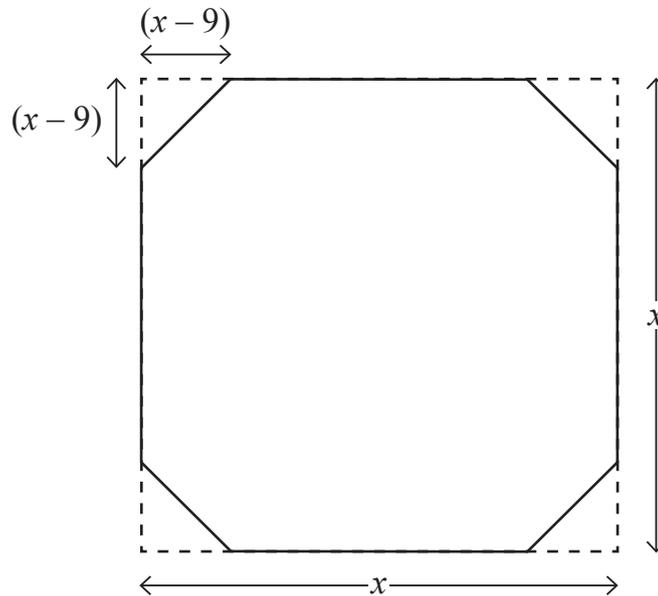
(i) Show that the total area of the four triangular corners is

$$2x^2 - 36x + 162$$

[1]

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The total area of the four triangular corners is $\frac{1}{8}$ of the area of the original square.

(ii) Find the value of x.

Answer _____ [6]

[Turn over

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(iii) Hence find the area of the octagonal plaque.

Answer _____ cm² [2]

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11 The curve C_1 is defined by the equation

$$y = -3x^2 - 4x + 1$$

(i) Find the equation of the line l which is the **tangent** to this curve at the point $(-1, 2)$.
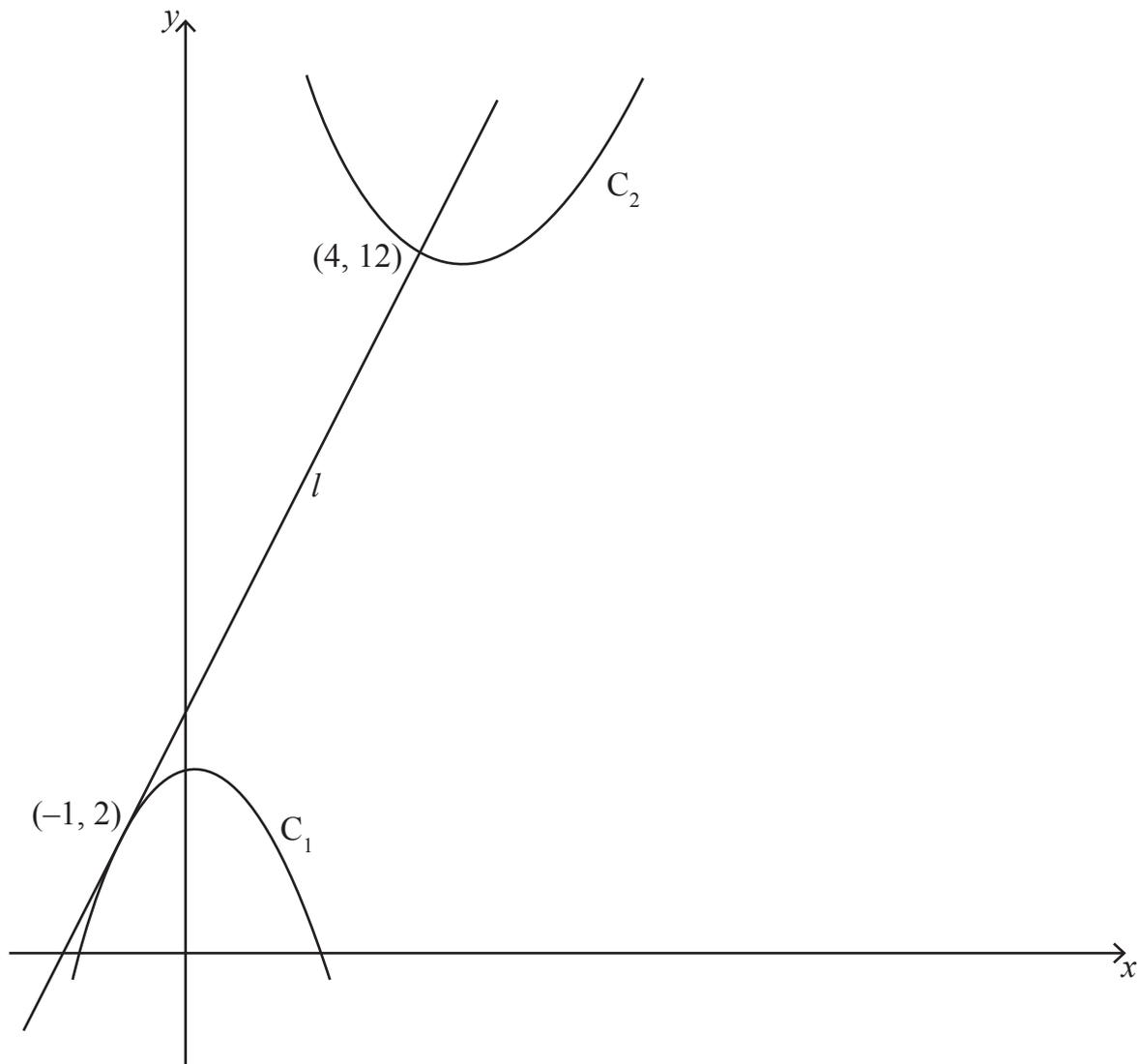
Answer _____ [3]

[Turn over

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The line l meets the curve C_2, which is defined by the equation

$$y = \tfrac{1}{2}x^2 - \tfrac{9}{2}x + 22$$

at the point (4, 12), as shown in the diagram below.

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(ii) Show that the equation of the **normal** to the curve C_2 at the point $(4, 12)$ is the same as the equation of the line l.
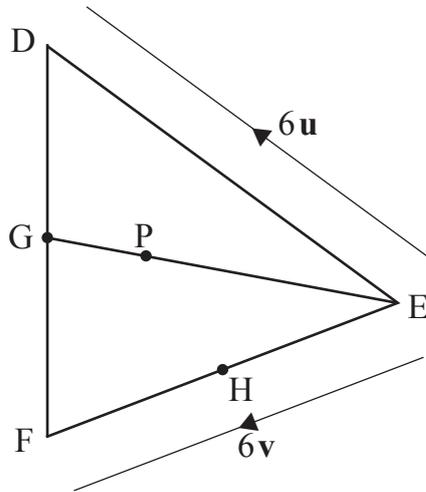
[3]

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[Turn over

12 In the triangle DEF shown below, G is the midpoint of DF and H is the midpoint of EF.

The vectors $\overrightarrow{ED}$ and $\overrightarrow{EF}$ are given by 6**u** and 6**v** respectively.

(i) Express each of the following vectors in terms of **u** and **v**, simplifying your answers as far as possible:

(a) $\overrightarrow{DF}$

Answer _____ [1]

(b) $\overrightarrow{EG}$

Answer _____ [2]

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(c) $\overrightarrow{DH}$

Answer _____ [2]

P is the point on the line EG such that $\overrightarrow{EP} = 2\overrightarrow{PG}$

(ii) Find the vector $\overrightarrow{HP}$

Answer _____ [3]

[Turn over

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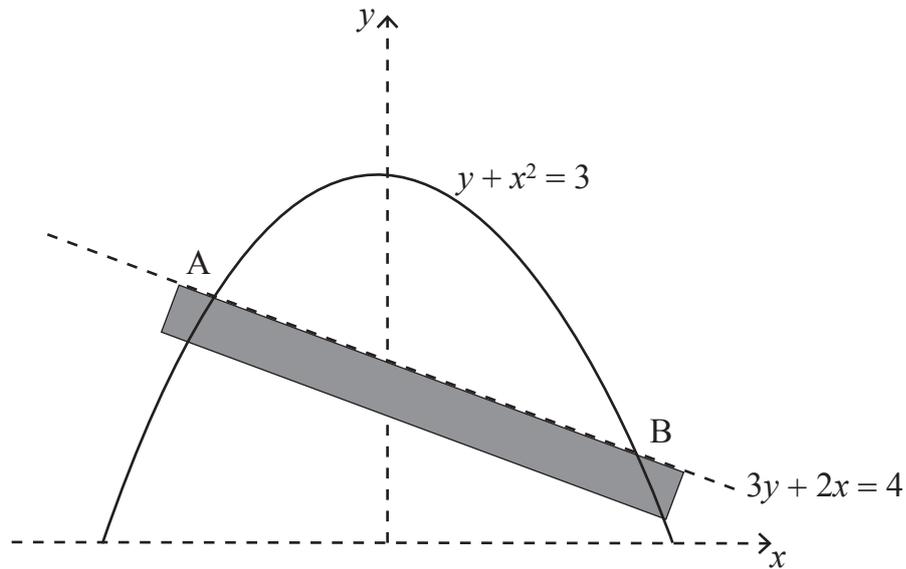**13** Maria is a set designer for a film production company. For one scene she must design a banner to hang across an archway.

She sets the banner so that the top edge can be modelled by a straight line with equation $3y + 2x = 4$

The archway can be modelled by the quadratic equation $y + x^2 = 3$

Derive a quadratic equation satisfied by x and hence determine the coordinates of the points A and B where the banner needs to be attached to the archway.
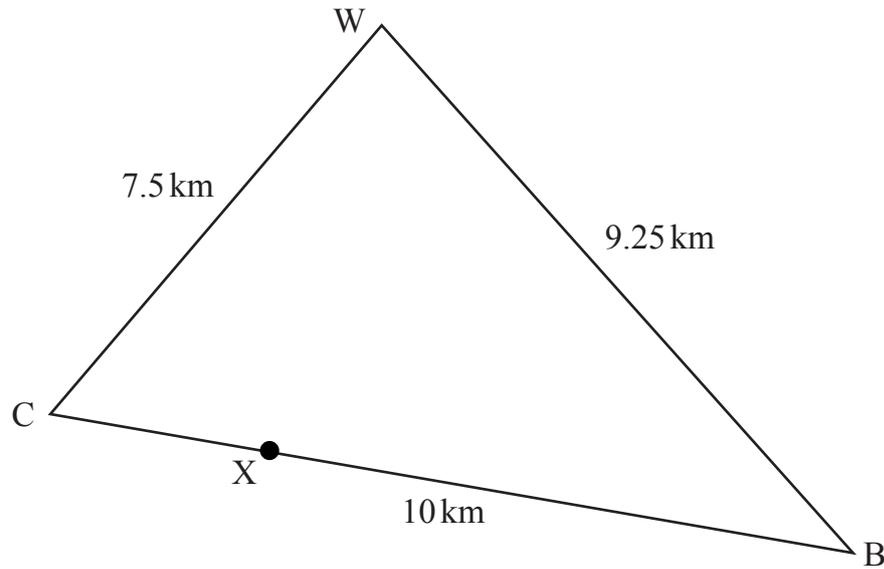
Answer _____ [6]

[Turn over

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14 The distance between Carrickfergus (C) and Bangor (B) is 10 km, between Carrickfergus and Whitehead (W) is 7.5 km and between Whitehead and Bangor is 9.25 km, as shown in the diagram below.

(i) Calculate the size of the angle WĈB.

Answer _____ ° [2]

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A boat leaves Carrickfergus at 2.00 pm and travels at a constant speed directly towards Bangor.

At 2.15 pm an observer at Whitehead spotted the boat at a point X, where $C\hat{W}X = 26.5°$

(ii) Calculate the size of the angle $C\hat{X}W$.

Answer _____ ° [1]

(iii) Calculate the distance CX.

Answer _____ km [2]

[Turn over

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(iv) Find the speed of the boat in km/h.

Answer _____ km/h [1]

(v) Find the time of arrival of the boat in Bangor to the nearest minute.

Answer _____ [1]

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(Questions continue overleaf)

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15 A curve is defined by the equation $y = x(x-2)^2$

(i) Write down the coordinates of the points where the curve meets the *x*-axis.

Answer _____ [2]

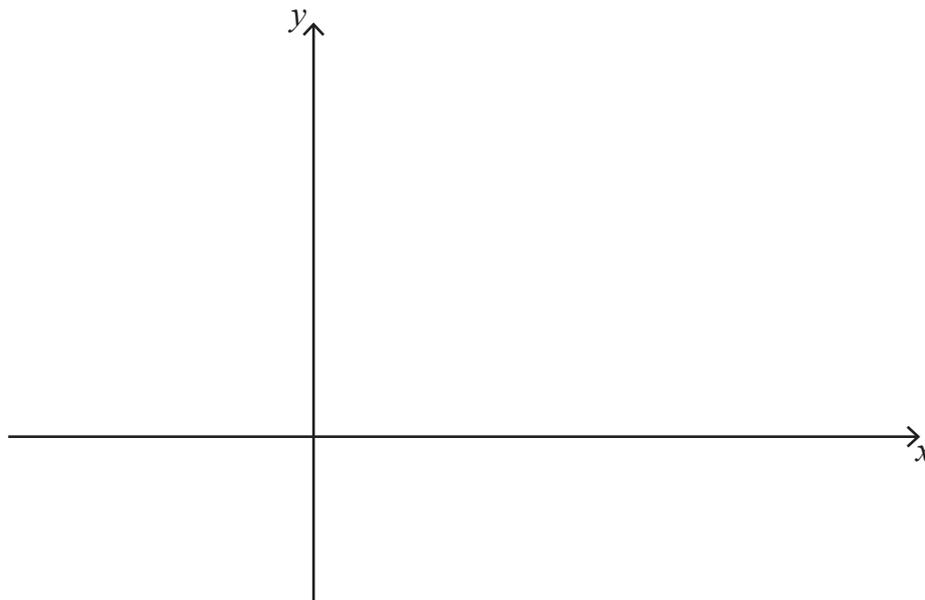**(ii)** Find the coordinates of the turning points of the curve.

Answer _____ [6]

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(iii) Using calculus, identify each turning point as either a maximum or a minimum point. You **must** show working to justify your answer.

Answer _____ [2]

(iv) Sketch the curve on the axes below.

y

x

[2]

[Turn over

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(v) Find the area enclosed by the curve and the *x*-axis.
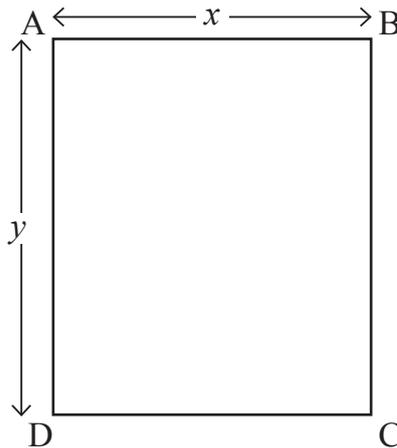
Answer _____ [3]

16 A builder wishes to fence off a rectangular plot of land ABCD.

For the three sides AD, DC and BC he can use fencing which costs £5 per metre.

For safety reasons he needs to use taller fencing, which costs £10 per metre, for the side AB.

Let x m and y m be the lengths of the sides AB and AD respectively, as shown in the diagram below.

(i) Write down, in terms of x and y, the total cost of the fencing.

Answer _____ [1]

[Turn over

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The builder wishes to spend a total of £600 on fencing.

(ii) Show that

$$y = 60 - \tfrac{3}{2}x$$

[2]

(iii) Hence express the area A of the plot in terms of x.

Answer _____ [1]

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(iv) Find the dimensions of the plot which give the maximum area, proving that it is a maximum.

Answer _____ [4]

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For Examiner's use only	
Question Number	**Marks**
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