Rewarding Learning

General Certificate of Secondary Education
2017

Further Mathematics

Unit 1
Pure Mathematics

[GMF11]

TUESDAY 13 JUNE, MORNING

MARK SCHEME

General Marking Instructions

Introduction

Mark schemes are published to assist teachers and students in their preparation for examinations. Through the mark schemes teachers and students will be able to see what examiners are looking for in response to questions and exactly where the marks have been awarded. The publishing of the mark schemes may help to show that examiners are not concerned about finding out what a student does not know but rather with rewarding students for what they do know.

The Purpose of Mark Schemes

Examination papers are set and revised by teams of examiners and revisers appointed by the Council. The teams of examiners and revisers include experienced teachers who are familiar with the level and standards expected of students in schools and colleges.

The job of the examiners is to set the questions and the mark schemes; and the job of the revisers is to review the questions and mark schemes commenting on a large range of issues about which they must be satisfied before the question papers and mark schemes are finalised.

The questions and the mark schemes are developed in association with each other so that the issues of differentiation and positive achievement can be addressed right from the start. Mark schemes, therefore, are regarded as part of an integral process which begins with the setting of questions and ends with the marking of the examination.

The main purpose of the mark scheme is to provide a uniform basis for the marking process so that all the markers are following exactly the same instructions and making the same judgements in so far as this is possible. Before marking begins a standardising meeting is held where all the markers are briefed using the mark scheme and samples of the students' work in the form of scripts. Consideration is also given at this stage to any comments on the operational papers received from teachers and their organisations. During this meeting, and up to and including the end of the marking, there is provision for amendments to be made to the mark scheme. What is published represents this final form of the mark scheme.

It is important to recognise that in some cases there may well be other correct responses which are equally acceptable to those published: the mark scheme can only cover those responses which emerged in the examination. There may also be instances where certain judgements may have to be left to the experience of the examiner, for example, where there is no absolute correct response – all teachers will be familiar with making such judgements.

			AVAILABLE MARKS
1	**(i)** $2\mathbf{P} - \mathbf{Q} = 2\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$		
	$= \begin{bmatrix} 6 & 2 \\ -2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 4 & 5 \\ -3 & 0 \end{bmatrix}$	MW1	
	(ii) $\mathbf{P}^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$	M1 W1	
	(iii) $\frac{1}{2}\mathbf{R} + \mathbf{Q} = \mathbf{P}^2$		
	$\frac{1}{2}\mathbf{R} = \mathbf{P}^2 - \mathbf{Q} = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 2 & -3 \\ 1 & 4 \end{bmatrix}$	MW1	
	$= \begin{bmatrix} 6 & 8 \\ -6 & -1 \end{bmatrix}$		
	$\mathbf{R} = \begin{bmatrix} 12 & 16 \\ -12 & -2 \end{bmatrix}$	W1	5
2	**(i)** $x + \frac{23}{x} = 10$		
	$x^2 + 23 = 10x$		
	$x^2 - 10x + 23 = 0$	MW1	
	(ii) $x + \frac{23}{x} = 10$		
	From **(i)**		
	$x^2 - 10x + 23 = 0$		
	$(x - 5)^2 - 25 + 23 = 0$	M1 W1	
	$(x - 5)^2 = 2$	W1	
	$x - 5 = \pm\sqrt{2}$		
	$x = 5 \pm \sqrt{2}$	W1	5

			AVAILABLE MARKS
3	$y = 3x - \frac{1}{2x^3}$		
	$= 3x - \frac{1}{2}x^{-3}$		
	$\frac{dy}{dx} = 3 + \frac{3}{2}x^{-4}$	MW1	
	$\frac{d^2y}{dx^2} = -\frac{12}{2}x^{-5} = -6x^{-5}$ or $-\frac{6}{x^5}$	MW1 MW1	3
4	$\int \frac{2}{3x^9}\,dx$		
	$= \int \frac{2}{3}x^{-9}\,dx$		
	$= \frac{2}{3}\left(\frac{x^{-8}}{-8}\right) + c$		
	$= -\frac{1}{12}x^{-8} + c$ or $-\frac{1}{12x^8} + c$	MW1 MW1	2

AVAILABLE MARKS

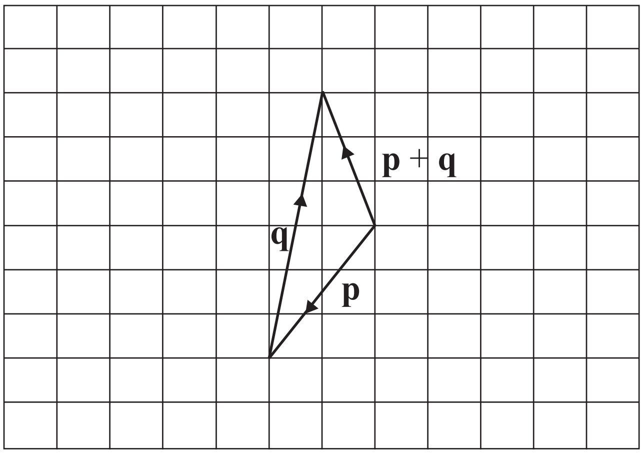
5 **(a)** $2\mathbf{x} + \begin{bmatrix} 4 \\ -2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} - 4\begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 4 \\ -8 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$

$2\mathbf{x} = \begin{bmatrix} 6 \\ -5 \end{bmatrix} - \begin{bmatrix} 4 \\ -2 \end{bmatrix}$ M1

$2\mathbf{x} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$

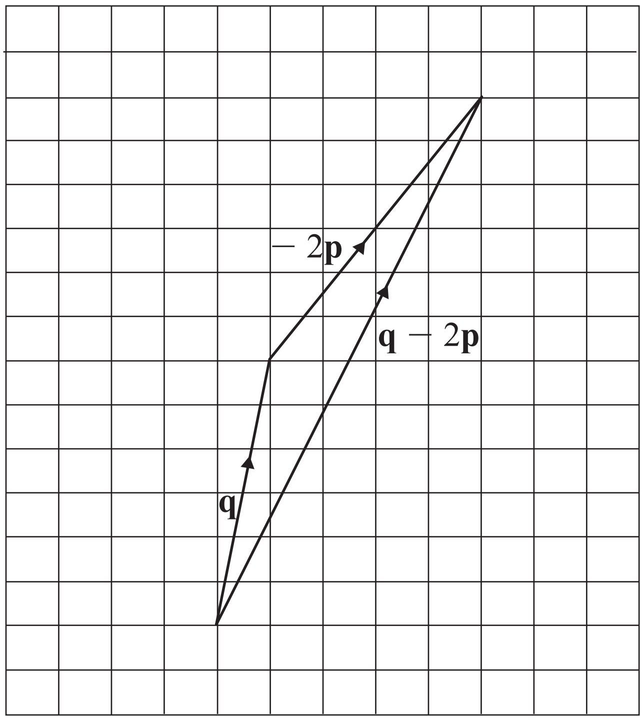
$\mathbf{x} = \begin{bmatrix} 1 \\ -1.5 \end{bmatrix}$ W1

(b) **(i)** **p** + **q**



W1

(ii) **q** - 2**p**



W1 4

		AVAILABLE MARKS
6 **(i)** $\tan\theta = 1.56$		
$\theta = 57.339°$ or $237.339°$		
$\rightarrow 57.34°$ or $237.34°$	MW1 MW1	
(ii) $\tan\left(\frac{5}{6}x - 45°\right) = 1.56$		
From **(i)** $\frac{5}{6}x - 45° = 57.339°$ or $237.339°$	M1	
$\frac{5}{6}x = 102.339°$ or $282.339°$		
$x = 122.81°$ or $338.81°$	W1 W1	5
7 $\mathbf{AX} = \mathbf{B}$		
$\mathbf{X} = \mathbf{A}^{-1}\mathbf{B}$		
$\mathbf{A}^{-1} = \frac{1}{10}\begin{bmatrix} 4 & -6 \\ -3 & 7 \end{bmatrix}$	M1 W1 W1	
$\mathbf{X} = \frac{1}{10}\begin{bmatrix} 4 & -6 \\ -3 & 7 \end{bmatrix}\begin{bmatrix} 19 & 33 \\ 11 & 17 \end{bmatrix}$	M1	
$= \frac{1}{10}\begin{bmatrix} 10 & 30 \\ 20 & 20 \end{bmatrix}$		
$= \begin{bmatrix} 1 & 3 \\ 2 & 2 \end{bmatrix}$	W1	5

			AVAILABLE MARKS
8	**(i)** $\dfrac{3(x-4)}{x^2-16}+\dfrac{2}{x-1}$		
	$=\dfrac{3(x-4)}{(x-4)(x+4)}+\dfrac{2}{x-1}$	MW1	
	$=\dfrac{3}{x+4}+\dfrac{2}{x-1}$	MW1	
	$=\dfrac{3(x-1)+2(x+4)}{(x+4)(x-1)}$	M1	
	$=\dfrac{5x+5}{(x+4)(x-1)}=\dfrac{5(x+1)}{(x+4)(x-1)}$	W1	
	(ii) $\dfrac{3(x-4)}{x^2-16}+\dfrac{2}{x-1}=\dfrac{5}{6}$		
	From **(i)**		
	$\dfrac{5(x+1)}{(x+4)(x-1)}=\dfrac{5}{6}$	M1	
	$6(x+1)=(x+4)(x-1)$	MW1	
	$6x+6=x^2+3x-4$		
	$x^2-3x-10=0$		
	$(x+2)(x-5)=0$		
	$x=-2$ or $x=5$	W1	7

9 **(i)**	$2\log 3x + \log 2 - \log 7$		AVAILABLE MARKS
	$= \log(3x)^2 + \log 2 - \log 7$		
	$= \log 9x^2 + \log 2 - \log 7$		
	$= \log\left(\frac{18x^2}{7}\right)$	MW1 MW1 MW1	
(ii)	$2\log_5 3x + \log_5 2 - \log_5 7 = 2$		
	From **(i)**		
	$\log_5\left(\frac{18x^2}{7}\right) = 2$		
	$\frac{18x^2}{7} = 5^2 = 25$	M1	
	$x^2 = \frac{7 \times 25}{18}$		
	$x = 3.12$	W1	5

			AVAILABLE MARKS
10	**(i)**	Total area of triangles $= 4 \times \frac{1}{2}(x - 9)(x - 9)$	MW1	
		$= 2x^2 - 36x + 162$		
	(ii)	Area of triangles $= \frac{1}{8}$ area of square		
		$2x^2 - 36x + 162 = \frac{1}{8}x^2$	M1 MW1	
		$16x^2 - 288x + 1296 = x^2$		
		$15x^2 - 288x + 1296 = 0$	W1	
		$5x^2 - 96x + 432 = 0$		
		$x = \dfrac{96 \pm \sqrt{96^2 - 4(5)(432)}}{10}$		
		[or $(5x - 36)(x - 12) = 0$]	M1	
		$x = 12$ or 7.2	W1	
		But $x \neq 7.2$ as sides of triangles would have negative length.		
		So $x = 12$	M1	
	(iii)	Octagonal area $= x^2 - (2x^2 - 36x + 162)$	M1	
		$= 144 - 18$		
		$= 126 \text{ cm}^2$	W1	9

[Alternatively:

Octagonal area $= \frac{7}{8}x^2$ M1

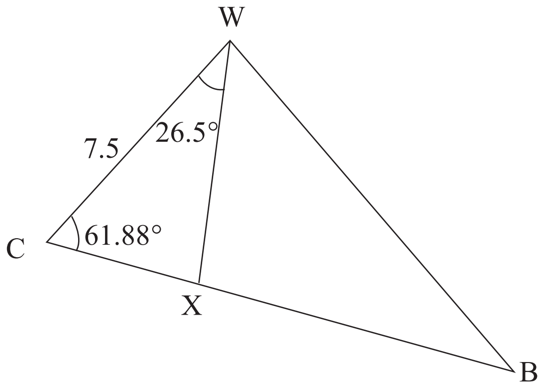
$= \frac{7}{8} \times 144$

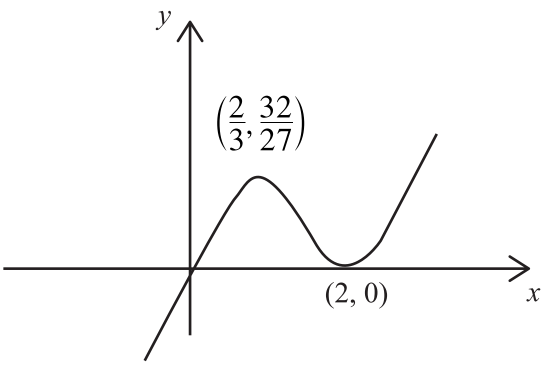
$= 126 \text{ cm}^2$ W1]

			AVAILABLE MARKS
11	**(i)**	$y = -3x^2 - 4x + 1$	
		$\frac{dy}{dx} = -6x - 4$ MW1	
		At $x = -1$, $\frac{dy}{dx} = 6 - 4 = 2$ W1	
		Equation of l is $y = 2x + c$	
		When $x = -1$, $y = 2$	
		$2 = -2 + c$	
		$\therefore c = 4$	
		Equation of tangent, l, is $y = 2x + 4$ MW1	
	(ii)	$y = \frac{1}{2}x^2 - \frac{9}{2}x + 22$	
		$\frac{dy}{dx} = x - \frac{9}{2}$ MW1	
		At (4, 12), $\frac{dy}{dx} = 4 - \frac{9}{2} = -\frac{1}{2}$	
		Gradient of normal at (4, 12) = 2 MW1	
		Equation of normal is $y = 2x + c$	
		When $x = 4$, $y = 12$	
		$12 = 8 + c$	
		$\therefore c = 4$	
		Equation of normal is $y = 2x + 4$ MW1	
		This is the equation of l	6

		AVAILABLE MARKS
12 (i) (a) $\overrightarrow{DF} = \overrightarrow{DE} + \overrightarrow{EF}$		
$= -6\mathbf{u} + 6\mathbf{v}$	MW1	
(b) $\overrightarrow{EG} = \overrightarrow{ED} + \frac{1}{2}\overrightarrow{DF}$ (**or** $\overrightarrow{EF} - \frac{1}{2}\overrightarrow{DF}$)		
$= 6\mathbf{u} + \frac{1}{2}(-6\mathbf{u} + 6\mathbf{v})$	M1	
$= 3\mathbf{u} + 3\mathbf{v}$	W1	
(c) $\overrightarrow{DH} = \overrightarrow{DE} + \frac{1}{2}\overrightarrow{EF}$ (**or** $\overrightarrow{DF} - \frac{1}{2}\overrightarrow{EF}$)		
$= -6\mathbf{u} + \frac{1}{2}(6\mathbf{v})$	M1	
$= -6\mathbf{u} + 3\mathbf{v}$	W1	
(ii) $\overrightarrow{EP} = \frac{2}{3}\overrightarrow{EG} = 2\mathbf{u} + 2\mathbf{v}$	MW1	
$\overrightarrow{HP} = \overrightarrow{HE} + \overrightarrow{EP}$		
$= \frac{1}{2}(-6\mathbf{v}) + (2\mathbf{u} + 2\mathbf{v})$	M1	
$= 2\mathbf{u} - \mathbf{v}$	W1	8

		AVAILABLE MARKS
13 $y + x^2 = 3$		
$y = 3 - x^2$	W1	
$3y + 2x = 4$		
$9 - 3x^2 + 2x = 4$	M1	
$3x^2 - 2x - 5 = 0$	W1	
Alternatively		
$3y + 2x = 4$		
$y = \frac{4 - 2x}{3}$	W1	
$y + x^2 = 3$		
$\frac{4 - 2x}{3} + x^2 = 3$	M1	
$4 - 2x + 3x^2 = 9$		
$3x^2 - 2x - 5 = 0$	W1	
$(3x - 5)(x + 1) = 0$	M1	
$x = \frac{5}{3}$ or -1		
$y = \frac{2}{9}$ or 2		
Points are A$(-1, 2)$ and B$\left(\frac{5}{3}, \frac{2}{9}\right)$	W2	6

			AVAILABLE MARKS
14	**(i)** $WB^2 = CW^2 + CB^2 - 2 . CW . CB \cos W\hat{C}B$		
	$9.25^2 = 7.5^2 + 10^2 - 2 \times 7.5 \times 10 \cos W\hat{C}B$	M1	
	$W\hat{C}B = 61.8845° \rightarrow 61.88°$	W1	
	(ii) 		
	$C\hat{X}W = 180° - (61.8845° + 26.5°)$		
	$= 91.6155° \rightarrow 91.62°$	MW1	
	(iii) $\dfrac{CX}{\sin C\hat{W}X} = \dfrac{CW}{\sin C\hat{X}W}$		
	$CX = \dfrac{7.5 \sin 26.5°}{\sin 91.6155°}$	M1	
	$= 3.348\,\text{km} \rightarrow 3.35\,\text{km}$	W1	
	(iv) Boat travels 3.348 km in 15 mins		
	Speed $= \dfrac{3.348}{0.25} = 13.392\,\text{km/h}$		
	$\rightarrow 13.39\,\text{km/h}$	MW1	
	(v) Time to travel distance CB $= \dfrac{10}{13.392}$		
	$= 0.747\,\text{h} = 45$ mins (to nearest min)	MW1	
	Time of arrival is 2.45 pm		7

			AVAILABLE MARKS
15 **(i)**	Meets the x-axis at $(0, 0)$ and $(2, 0)$	MW1 MW1	
(ii)	$y = x(x^2 - 4x + 4) = x^3 - 4x^2 + 4x$	MW1	
	$\frac{dy}{dx} = 3x^2 - 8x + 4 = 0$	M1 W1 M1	
	$(3x - 2)(x - 2) = 0$		
	$x = \frac{2}{3}$ or $x = 2$		
	$y = \left(\frac{2}{3}\right)^3 - 4\left(\frac{2}{3}\right)^2 + 4\left(\frac{2}{3}\right) = \frac{32}{27}$		
	or $y = 2(0) = 0$		
	Turning points at $\left(\frac{2}{3}, \frac{32}{27}\right)$ and $(2, 0)$	W2	
(iii)	$\frac{d^2y}{dx^2} = 6x - 8$	MW1	
	When $x = \frac{2}{3}, \frac{d^2y}{dx^2} = -4 < 0$		
	Maximum at $\left(\frac{2}{3}, \frac{32}{27}\right)$		
	When $x = 2, \frac{d^2y}{dx^2} = 4 > 0$		
	Minimum at $(2, 0)$	MW1	
(iv)		M1 (shape) W1 (points)	
(v)	Area $= \int_0^2 (x^3 - 4x^2 + 4x)dx$	M1	
	$= \left[\frac{x^4}{4} - \frac{4}{3}x^3 + 2x^2\right]_0^2$	MW1	
	$= \left[4 - \frac{32}{3} + 8\right] - [0]$		
	$= 1\frac{1}{3}$	W1	15

		AVAILABLE MARKS
16 **(i)** Total cost $= 10x + 5y + 5x + 5y$		
$= 15x + 10y$	MW1	
(ii) $15x + 10y = 600$	M1	
$10y = 600 - 15x$	MW1	
$y = 60 - \frac{3}{2}x$		
(iii) $A = x\,y = x\left(60 - \frac{3}{2}x\right)$		
$= 60\,x - \frac{3}{2}x^2$	MW1	
(iv) For maximum area, $\frac{dA}{dx} = 0$		
$\frac{dA}{dx} = 60 - 3x = 0$	MW1	
$\therefore x = 20$	W1	
$\frac{d^2A}{dx^2} = -3 < 0 \therefore$ maximum	MW1	
$y = 60 - \frac{3}{2}(20) = 30$	W1	
So plot has dimensions 20 m × 30 m		8
	Total	**100**